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FRICTION FACTOR DETERMINATION IN DIFFERENT REGIONS OF THE MOODY CHART

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1 ABSTRACT

Calculating pressure loss (hf) in pipes is essential for sizing pressurized irrigation systems. The Darcy-Weisbach equation is the most recommended method for estimating hf, and it requires the calculation of the friction factor, f. The hypothesis of this research is that general equations for calculating f, which cover the entire Moody diagram, do not always produce results sufficiently close to those obtained from the specific equations for each flow regime. The objective of this study was to evaluate the performance of general f estimation equations across different regions of the Moody diagram (i.e., different flow regimes) and, based on the results, to develop a MATLAB code for its calculation. The results confirmed the hypothesis that general equations for calculating f do not always yield results sufficiently close to those derived from the specific equations for each flow regime. The mean relative error (MRE) was used to measure the accuracy of the equations. Based on the smallest MRE, the following equations were selected for calculating f according to the flow regime: Swamee for the laminar regime, Konakov for the turbulent/smooth regime, Vatankhah for the turbulent/rough and transitional regimes, and the Offor & Alabi equation for the transient turbulent regime. A MATLAB code was developed using these equations to facilitate personal use or integration into software.
Keywords: irrigation systems; head loss; hydraulic.

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2 RESUMO

O cálculo da perda de carga (hf) em tubulações é fundamental para o dimensionamento de sistemas de irrigação pressurizados. A equação de Darcy-Weisbach é a mais recomendada para estimar hf e requer o cálculo do fator de atrito, f. A hipótese desta pesquisa é que as equações gerais para cálculo de f, que abrangem todo o Diagrama de Moody, nem sempre produzem resultados suficientemente próximos daqueles obtidos utilizando as equações específicas para

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cada regime de fluxo. O objetivo deste trabalho foi avaliar o desempenho de equações gerais de estimativa de f nas diferentes regiões do diagrama Moody (regimes de fluxo) e, com base nos resultados, desenvolver um código MATLAB para seu cálculo. Os resultados obtidos confirmaram a hipótese de que as equações gerais de cálculo de f nem sempre produzem resultados suficientemente próximos aos obtidos através das equações específicas quando aplicadas a cada regime de escoamento, tendo utilizado o erro relativo médio (MRE) para medir o ajuste. Com base nos menores MRE, as seguintes equações foram selecionadas para calcular f conforme o regime de fluxo: Swamee para o regime laminar, Konakov para o regime turbulento/suave, Vatankhah para o regime turbulento/áspero e o regime de transição, e a equação de Offor & Alabi para o regime turbulento transitório. Um código MATLAB foi desenvolvido usando essas equações para facilitar o uso pessoal ou para ser incorporado em software.

Keywords: perda de carga; fator de atrito; irrigação.

3 INTRODUCTION

Head loss is the reduction in pressure or energy of water as it flows through pipes and other components of a hydraulic network (e.g., an irrigation system), caused by friction and other resistive forces. It is influenced by factors such as water velocity, pipe length, pipe diameter, and pipe roughness.

Calculating pressure head loss is fundamental when sizing pressurized irrigation systems, as it affects uniformity, fixed costs associated with pipe diameters in the hydraulic network, and variable costs, particularly energy consumption.

The Darcy–Weisbach equation (Equation 1) is the most recommended equation for determining head loss in pipes (Brown, 2003) because of its theoretical basis and amplitude of application (Liou, 1998). It presents a friction coefficient (f) whose determination in an explicit and precise way is continuously studied and improved.

$$hf = \frac{8.f.L.Q^2}{\pi^2.g.D^5} \tag{1}$$

Where hf = head loss, mca; f = friction factor; L = pipeline length, m; Q = flow rate in the pipe, $m^3.s^{-1}$; g = acceleration of gravity, $m.s^{-2}$; and D = inside diameter of the pipe, m.

The applicability of the Darcy–Weisbach equation depends on how the friction factor (f) is calculated, which varies with the flow regime, the fluid in question, and the pipe roughness.

The Moody chart is a graphical tool used to estimate the friction factor for all flow regimes, which are classified as laminar, transitional, and turbulent. The turbulent flow regime is further subdivided into smooth pipe flow, transitional rough pipe flow, and rough pipe flow. However, the Moody chart is not ideal for computer-based simulations (Offor; Alabi, 2016).

Several explicit equations have been developed to estimate f in certain specific ranges of Moody's diagram, which represent a flow regime or part of this regime (Laviolette, 2017).

For laminar flow, the Hagen–Poiseuille equation is used, which depends solely on the Reynolds number (Re) (Romeo, Royo; Monzón, 2002).

For turbulent flow, the recommended equation for estimating the friction factor (f) is the Colebrook equation (Colebrook, 1939), which depends on both the Reynolds number (Re) and the relative roughness (e/D). The Colebrook equation is more complex because it is implicit. Owing to the complexity of solving the Colebrook equation (Mendes, 2024), several explicit equations have been developed by different authors. Among these equations are the Konakov equation, cited by Robaina (1992), for hydraulically smooth tubes, and the Nikuradse equation, cited by Allen (1996), for hydraulically rough tubes.

The development of general equations that can be applied over wide ranges or even across the entire Moody diagram, making them highly convenient for users, is another important area of research for determining the friction factor (Swamee; Jain, 1976; Swamee, 1993, Offor; Alabi, 2016; Mikata; Walczak, 2016; Vatankhah, 2018; Zeghadnia; Robert; Achour, 2019; Minhoni *et al*., 2020).

Pimenta *et al.* (2018) listed 29 general explicit equations for determining the friction factor f, with the equation by Offor and Alabi (2016) being the most accurate, with a maximum relative error of 2.128%. This makes it a viable alternative to the Colebrook equation. Vatankhah (2018) presented a simpler equation with greater precision than the Offor and Alabi equation, achieving a maximum error of less than 0.054%.

However, the analysis of the general explicit equations was limited to their overall performance across all flow regimes, without evaluating their efficacy in each individual regime.

The research problem can be summarized by the following question: When analyzing each flow regime, do the general equations for calculating f provide approximately the same level of precision as the specific equations?

The hypothesis of this research is that the general equations for calculating the friction factor (f) do not always produce results that are sufficiently close to those obtained via the specific equations for each flow regime, with the mean relative error (MRE) used to measure the accuracy of the results.

The objective of this work was to evaluate the performance of the general equation in estimating the friction factor (f) across the different regions of the Moody diagram (flow regimes) and, on the basis of the results, develop a MATLAB code for its calculation.

4 MATERIALS AND METHODS

The Moody diagram was divided into different regions to characterize the flow regimes (Table 1). The specific equation recommended for each region was obtained from the literature (Table 2) to calculate the friction factor (f) in the Darcy–Weisbach equation.

Table 1. Flow regime according to the Reynolds number (Re) and the D/e ratio.

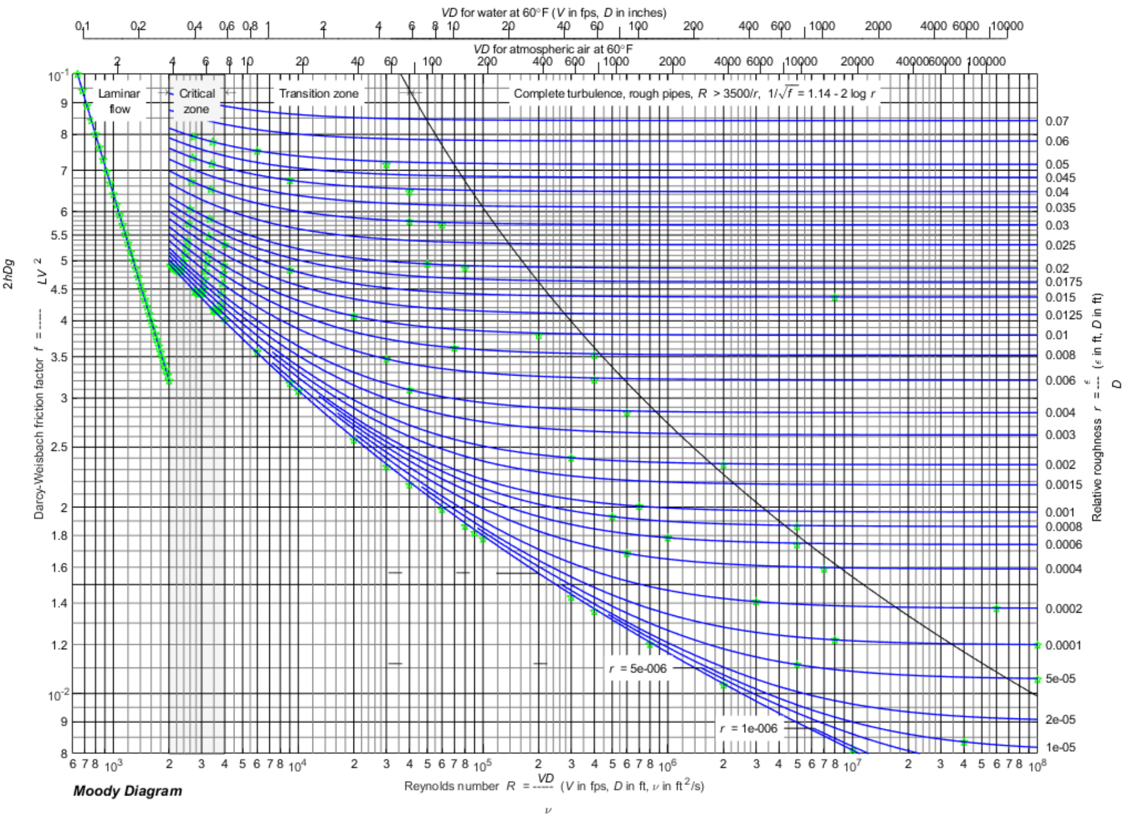
Criterion	**Flow regime**
$Re < 2000$	Laminar
$2000 \leq Re \leq 4000$	Transitional
$Re > 4000$	Turbulent
$Re > 4000$ and $\frac{Re}{D/e}^{0.9} < 31$	Turbulent flow in smooth pipe
$Re > 4000$ e $31 < \frac{Re}{D/e}^{0.9} < 448$	Turbulent flow in transitional rough pipe
$Re > 4000$ e $\frac{Re}{D/e}^{0.9} \geq 448$	Turbulent flow in rough pipe

Table 2. Specific equations for each region of the Moody diagram (or for the flow regimes).

Flow regime	Equation		Author
Laminar	$f = \frac{64}{Re}$	(2)	Hagen-Poiseuille
Transitional and Turbulent transitional	$\frac{1}{\sqrt{f}} = -2 \log\left(\frac{2.51}{Re.\sqrt{f}} + \frac{e}{3.71D}\right)$	(3)	Colebrook
Turbulent, smooth pipe	$f = \left[-2 \log\left(\frac{5.62}{Re^{0.9}}\right)\right]^{-2}$	(4)	Konakov
Turbulent, rough pipe	$f = \left[-2 \log\left(\frac{e}{3.71\ D}\right)\right]^{-2}$	(5)	Nikuradse

A total of 163 f values, covering the entire Moody diagram and representing combinations of the Reynolds number (Re) and relative roughness (ϵ/D), were used to evaluate the performance of the general explicit equations for calculating the friction factor in each flow regime. Each point was calculated via a specific equation (Table 2) corresponding to the region of the diagram (flow regime) and Microsoft Excel spreadsheets. These points were used as the reference for comparison and are highlighted in green in Figure 1.

Figure 1. The Moody chart shows the points (highlighted in green) used as references for evaluating the general equations. The figure was generated using MATLAB code adapted from Lopez (2019).



The general equations for estimating the friction factor (f), which are applicable to all regions of the Moody diagram, are presented in Table 3. These equations were selected on the basis of their widespread use in the design of irrigation systems (Swamee and Swamee & Jain equations) and/or their high precision (Vatankhak and Offor & Alabi equations).

Table 3. General explicit equations for estimating the friction factor (f) in the Darcy–Weisbach equation.

Author	Equation
Swamee and Jain (1976)	$\frac{1}{\sqrt{f}} = -2\log\left(\frac{e}{3.7\,D} + \frac{5.74}{Re^{0.9}}\right)$ (6)
Swamee (1993)	$f = \left\{\left(\frac{64}{Re}\right)^{8} + 9.5\left[\ln\left(\frac{e}{3.7\,D} + \frac{5.74}{Re^{0.9}}\right) - \left(\frac{2500}{Re}\right)^{6}\right]^{-16}\right\}^{0.125}$ (7)
Offor and Alabi (2016)	$\frac{1}{\sqrt{f}} = -2\log\left\{\frac{e}{3.71D} - \frac{1.975}{Re}\left[\ln\left(\left(\frac{e}{3.93D}\right)^{1.092} + \left(\frac{7.627}{Re + 395.9}\right)\right)\right]\right\}$ (8)
Vatankhah (2018)	$\frac{1}{\sqrt{f}} = 0.8686\ln\left[\frac{0.3984Re}{(0.8686s)^{\frac{s-0.645}{s+0.39}}}\right]$ (9) $where \quad s = 0.12363\,Re\left(\frac{e}{D}\right) + \ln(0.3984\,Re).$ (10)

The mean relative error (MRE) was used to evaluate the accuracy of the general equations (Table 3) in estimating the friction factor (f) in comparison with the 163 reference points (Figure 1) obtained with specific equations (Table 1). Sadeghi, Peters e Lamm (2015) identified the MRE as a useful parameter for evaluating the most accurate model for estimating f in each analyzed flow regime.

The MRE is calculated as follows:

$$MRE = \frac{estimated\ value - reference\ value}{reference\ value} \quad (11)$$

The classification presented by Pimenta *et al.* (2018) was used to assess the quality of the fit, as measured by the MRE and described in Table 4.

Table 4. Mean relative error (MRE) classification.

Criterion	Classification
MRE ≤ 0.55%	Very good
0.55% < MRE ≤ 1%	Good
1% < MRE ≤ 2%	Average
2% < MRE ≤ 3%	Weak
MRE > 3%	Poor

Based on the results obtained, the best equations were identified for each section of the Moody diagram. These equations were adopted in the development of the MATLAB® code (Appendix 1) to calculate the friction factor for the Darcy–Weisbach equation.

5 RESULTS AND DISCUSSION

Using the MRE classification, Table 5 shows the results of the comparison between the general and specific equations. None of the general equations achieved a 'very good' or 'good' classification for all the evaluated flow regimes.

The Swamee & Jain equation yielded the poorest results, achieving a 'good' classification only for turbulent rough pipe flow. The Swamee equation, on the other hand, provided a 'very good' estimation for laminar flow and a 'good' classification for turbulent rough pipe flow.

The Vatankhak and Offor & Alabi equations demonstrated similar performance, achieving the best results and earning a 'very good' classification for transitional, turbulent transitional, and turbulent rough pipe flows. However, for laminar flow and turbulent flow in smooth pipes, the Vatankhak equation exhibited mean relative errors (MRE) of -11.26% and -7.59%, respectively, while the Offor & Alabi equation had MREs of -12.97% and -7.60%. Both equations were classified as 'poor' adjustments compared to the specific Hagen–Poiseuille and Konakov equations.

Pimenta *et al*. (2018), in their evaluation of 29 general explicit equations for determining the friction factor (f), reported that the Offor and Alabi (2016) equation was the most accurate, with a maximum relative error (MRE) of 2.128% across the entire Moody diagram. This value represents the average error across all flow regimes. However, this manuscript shows that the average value masks poor estimation performance for laminar flow, where the MRE was -12.97%. Excluding this value for the laminar regime, the average MRE for the Offor & Alabi equation would be 2.05%, which is consistent with the value reported by Pimenta *et al.* (2018).

Table 5. Mean relative error (MRE) of the general explicit equations for each flow regime.

Flow regime	**Especific Equation**	**MEAN RELATIVE ERROR (MRE) %**			
		General explicit equations			
		Vatankhak	**Offor&Alabi**	**Swamee&Jain**	**Swamee**
Laminar	Hagen-Poiseuille	-11.264 (poor)*	-12.974 (poor)	-19.457 (poor)	-0.004 (very good)
Transitional	Colebrook	0.014 (very good)	-0.077 (very good)	-3.010 (poor)	14.460 (poor)
Turbulent, smooth pipe	Konakov	-7.593 (poor)	-7.596 (poor)	-8.327 (poor)	-8.025 (poor)
Turbulent, transitional	Colebrook	0.020 (very good)	0.005 (very good)	-1.077 (average)	-1.024 (average)
Turbulent, rough pipe	Nikuradse	-0.510 (very good)	-0.52 (very good)	-0.895 (good)	-0.859 (good)

* Classification according to MRE (Pimenta *et al.*, 2018)

The Vatankhak equation shows a similar performance pattern. It produces an MRE of -11.264% for laminar flow and -7.593% for turbulent flow in smooth pipes, whereas Vatankhak (2018) reported a maximum error of less than 0.054% as the average across all flow regimes.

For laminar flow, the Swamee equation provides an accurate adjustment to the specific Hagen–Poiseuille equation, with a mean relative error (MRE) of -0.004%, classified as "very good". This is justified by the inclusion of the Hagen–Poiseuille formulation within the Swamee equation (Swamee; Swamee, 2007).

In the turbulent regime for hydraulically smooth pipes, all the general equations showed MREs in the range of 7.5% to 8.3%, indicating a 'poor' adjustment. In this case, the specific Konakov equation is more appropriate.

For transitional and turbulent transitional flows, the general equations recommended are Vatankhak and Offor & Alabi, which produce MRE values lower than 0.77%, both of which are classified as 'Very Good.'

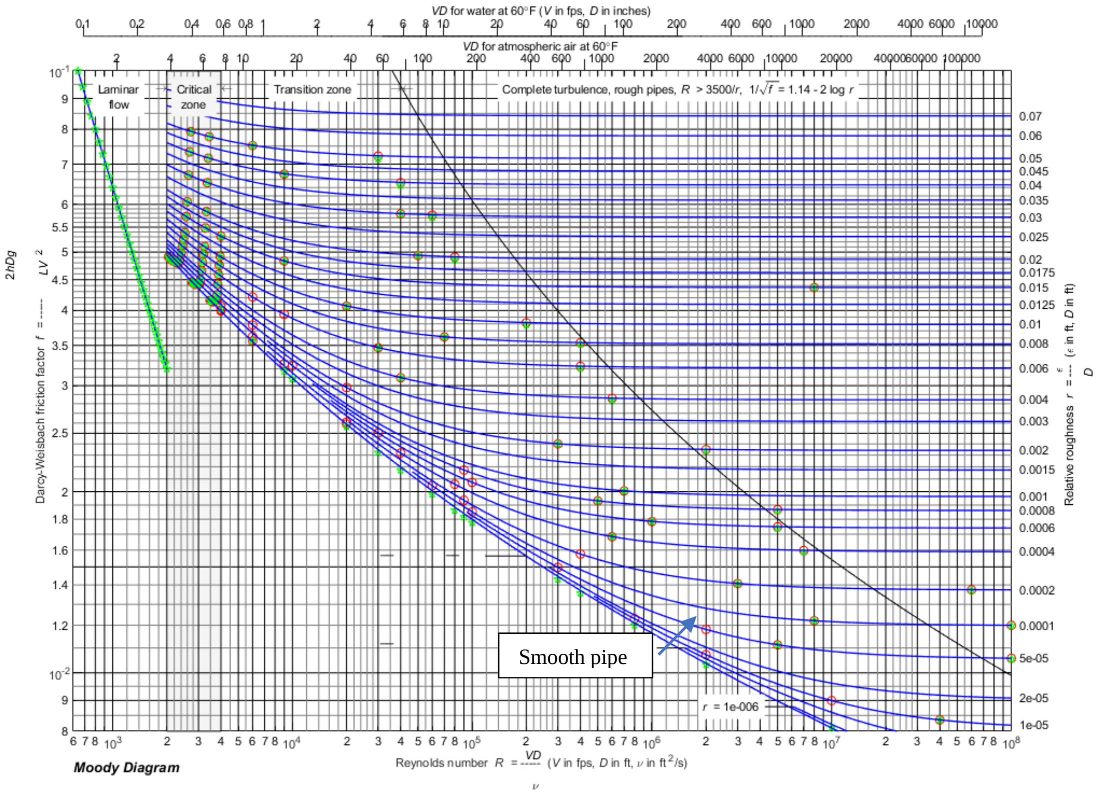
For turbulent flow in rough pipes, all the general equations can be recommended, with the Vatankhak and Offor & Alabi equations receiving a very good classification and the Swamee & Jain and Jain equations receiving a good classification.

The results obtained confirmed the hypothesis that the general equations for calculating the friction factor (f) do not always produce results that are sufficiently close to those obtained via the specific equations when applied to each flow regime, with the mean relative error (MRE) used to measure the adjustment.

Using the MRE as a performance indicator, the equations selected to compile the MATLAB code were as follows: Swamee for laminar flow, Konakov for turbulent flow in smooth pipes, Vatankhak for transitional and turbulent flow in rough pipes, and Offor & Alabi for transitional turbulent flow.

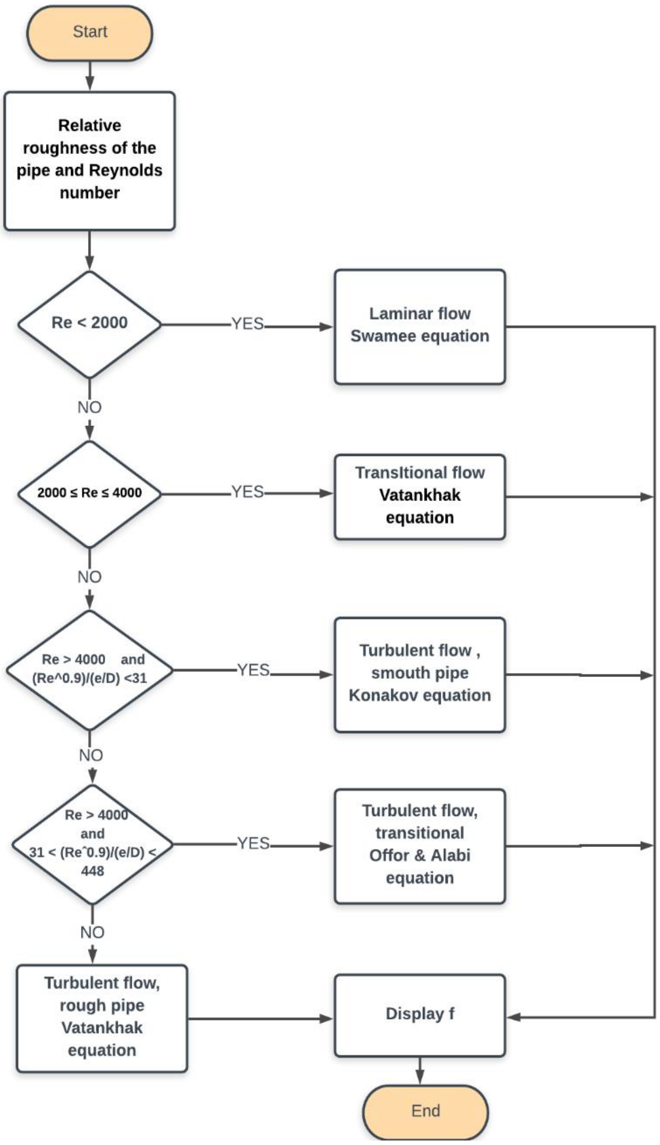
Figure 2 shows the Moody diagram with values estimated using the specific equation (Table 2) in green and values estimated using the best general equation for each flow regime, indicated in red. These points were obtained using the equation that provided the best performance in each section of the graph. The discrepancy between the specific equation and the general equation is evident in the turbulent flow in the smooth pipe, where the green and red points do not coincide.

Figure 2. The original points (in green) and the best estimates (in red) are represented in the Moody diagram. The figure was generated using MATLAB code adapted from Brandon Lopez (2019).



A flowchart (Figure 3) was generated from the equations selected for each region of the Moody diagram, which guided the development of the MATLAB code (Appendix 1) to calculate the factor f of the Darcy–Weisbach equation.

Figure 3. Flowchart of the friction factor (f) estimation used in the MATLAB code.



6 CONCLUSIONS

The results obtained confirmed the hypothesis that the general equations for calculating the friction factor (f) do not always produce results that are sufficiently close to those obtained via the specific equations when applied to each flow regime, with the mean relative error (MRE) used to measure the adjustment.

Based on the smallest relative errors obtained, the following equations were selected to calculate the friction factor for the Darcy–Weisbach equation according to the flow regime: Swamee for the laminar regime, Konakov for the turbulent/smooth regime, Vatankhah for the turbulent/rough regime and the transition regime, and the Offor & Alabi equation for the transient turbulent regime. A MATLAB code was developed using these equations to facilitate personal use or integration into software.

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8 ATTACHMENT – MATLAB Code

Code MATLAB

```
clear all;
close all;
clc
fprintf('\n\n "FRICTION FACTOR DETERMINATION"\n\n');
E=input('Enter with the relative roughness of the pipe (e/D):');
Re=input ('Enter with the Reynolds number (Re):');
if Re<=2000; disp ('Laminar flow')
   f = (((64/Re)^8)+9.5*(log((E/(3.7))+(5.74/(Re^0.9)))-((2500/Re)^6))^(-16))^0.125;
end
if Re>2000 && Re<4000; disp (‘Transitional flow’)
   S=(0.12363*Re*E)+(log(0.3984*Re));
   
f=(1/(0.8686*log((0.3984*Re)/((0.8686*S)^((S-0.645)/(S+0.39))))))^2;
end
if Re>=4000 && (Re^0.9)*(E)<=31;
   f=(-2*(log10(5.62/(Re^0.9))))^-2; disp ('Turbulent flow in smooth pipe')
end
if Re>=4000 && (Re^0.9)*(E)>=448;
   S=(0.12363*Re*E)+(log(0.3984*Re));
   
f=(1/(0.8686*log((0.3984*Re)/((0.8686*S)^((S-0.645)/(S+0.39))))))^2; disp ('Turbulent flow in rough pipe')
end
if Re>=4000 && ((Re^0.9)*(E))>31 && 448>((Re^0.9)*(E));
   f=(1/(-2*log10(((E/3.71)-((1.975/Re)*(log(((E/3.93)^1.092)+(7.627/(Re+395.9))))))))^2; disp ('Turbulent flow, transitional')
end
 disp ('Friction factor’);
```