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PRODUCTION FUNCTION AND YIELD OF SUGAR CANE CULTIVATED UNDER DIFFERENT WATER, NUTRITIONAL AND SUBSOILING MANAGEMENTS [1]

WANDERSON JOSÉ DE OLIVEIRA [2]; MARIO MONTEIRO ROLIM [3]; GERONIMO FERREIRA DA SILVA [4]; MANASSÉS MESQUITA DA SILVA [5]; CERES DUARTE GUEDES CABRAL DE ALMEIDA [6] AND EVANILSON PAULINO DA SILVA [7]

[1] Part of the first author's doctoral thesis, presented to the Postgraduate Program in Agricultural Engineering, Universidade Federal Rural de Pernambuco, Campus Recife, Recife PE, Brazil.
[2] Pernambuco Department of Education, Government of the State of Pernambuco, Av. Afonso Olindense, n° 1513, CEP - 50810-000, Recife PE, Brazil, wanderson.jdoliveira@professor.educacao.pe.gov.br
[3] Department of Agricultural Engineering, Federal Rural University of Pernambuco, Rua Dom Manoel de Medeiros, s/n, Dois Irmãos, CEP – 52171-900, Recife PE, Brazil, mario.rolim@ufrpe.br
evanilson.psilva@gmail.com,
[4] Department of Agricultural Engineering, Federal Rural University of Pernambuco, Rua Dom Manoel de Medeiros, s/n, Dois Irmãos, CEP – 52171-900, Recife PE, Brazil, g eronimo.silva@ufrpe.br
[5] Department of Agricultural Engineering, Federal Rural University of Pernambuco, Rua Dom Manoel de Medeiros, s/n, Dois Irmãos, CEP – 52171-900, Recife PE, Brazil, manasses.mesquita@ufrpe.br
[6] Colégio Dom Agostinho Ikas, Federal Rural University of Pernambuco, Rodovia PE 05 km 25, n° 4000, Tiúma, CEP - 54.737-200, Lourenço da Mata PE, Brazil, ceres.cabral@ufrpe.br
[7] Carpina Sugarcane Experimental Station, Federal Rural University of Pernambuco, Rua Ângela Cristina Canto Pessoa de Luna, s/n, Bairro Novo, CEP - 55812-010, Carpina PE, evanilson.psilva@gmail.com

1 RESUMO

Com o objetivo de identificar a função de produção que estima o máximo rendimento de açúcar e álcool na cana-de-açúcar cultivada com diferentes doses de nitrogênio, lâminas de irrigação em solo com e sem subsolagem, foi realizado um experimento em campo com cana soca. Os tratamentos consistiram em quatro lâminas de irrigação (0, 50, 100 e 150% da ETC) e 5 doses de N (0, 40, 80, 120 e 160 kg ha^{-1}). Foi utilizado um delineamento de 5x4x2 com 3 repetições em blocos inteiramente casualizados. A produtividade foi obtida por meio da pesagem dos colmos. A partir da função de produção foram determinadas a região de produção máxima, a taxa marginal de substituição e a região de produção racional. Através da produtividade foram calculados os rendimentos de açúcar e álcool. As lâminas, doses de N e produtividades máximas encontradas foram, de 2033 mm, 131 Mg ha^{-1} e 125 Mg ha^{-1} no solo sem subsolagem e de 1729 mm, 137 Mg ha^{-1} e 127 Mg ha^{-1} no solo com subsolagem. Para o maior rendimento de açúcar e álcool a aplicação de insumos deve permanecer nas doses de N entre 80 a 160 kg ha^{-1} e lâminas entre 1456 e 1731 mm.

Palavras-chave: curva de isoproduto, *Saccharum officinarum*, taxa marginal, álcool.

OLIVEIRA, W. J; ROLIM, M. M.; SILVA, G. F.; SILVA, M. M.; ALMEIDA, C. D. G.C. ; SILVA, E. P.

FUNCTION OF PRODUCTION AND YIELD IN SUGAR CANE CULTIVATED UNDER DIFFERENT MANAGEMENT NUTRITIONAL AND WATER AND SUBSOLATION

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2 ABSTRACT

With the objective of identifying the production function that estimates the maximum yield of sugar and alcohol in sugarcane cultivated with different doses of nitrogen and irrigation depths in soil with and without subsoiling, an experiment was carried out in the field with ratoon cane. Treatments consisted of four irrigation depths (0, 50, 100 and 150% of ETC) and 5 N doses (0, 40, 80, 120 and 160 kg ha-1). A 5x4x2 design with 3 replications in completely randomized blocks was used. Productivity was obtained by weighing the stalks. From the production function, the region of maximum production, the marginal rate of substitution and the region of rational production were determined. Through productivity, the yields of sugar and alcohol were calculated. The depths, N doses and maximum productivity found were 2033 mm, 131 Mg ha^{-1} and 125 Mg ha^{-1} in the soil without subsoiling and 1729 mm, 137 Mg ha^{-1} and 127 Mg ha^{-1} in the soil with subsoiling. For the highest yield of sugar and alcohol, the application of inputs should remain at doses of N between 80 and 160 kg ha^{-1} and depths between 1456 and 1731 mm.

Keywords: y field function, isoproduct curve, *Saccharum officinarum,* marginal rate, alcohol.

3 INTRODUCTION

Sugarcane is a crop that has great economic importance in Brazil due to the large production of sugar and ethanol. In 2020, sugarcane production in the world was approximately 1.9 billion tons, with more than 50% of production concentrated in the Americas, with Brazil being the largest producer in the world (FAOSTAT, 2022).

In Brazil, sugarcane is produced mainly under rainfed conditions. Thus, water stress is the main factor that harms crop production, especially in the initial budding phase and during the intense vegetative growth phase (INMAN-BAMBER, 2004; BASNAYAKE), 2015 *et al)* .

In addition to water deficit, several other factors stand out as limiting production, such as nitrogen (N) deficiency. Cunha *et al.* (2016) and Nascimento *et al.* (2018), when studying the yield of sugarcane subjected to nitrogen fertilization, found that the application of nitrogen promoted greater plant development and greater alcohol yield. Another limiting factor in sugarcane production in Brazil is soil compaction, as it directly interferes with water availability and nutrient absorption by the crop. Furthermore, it causes less root development, which is directly reflected in less water and nutrient absorption by plants and, consequently, promotes a decrease in the yield of ratoon sugarcane (2012; CHERUBIN., 2016 ALAMEDA; ANTEN,).

Sugarcane production aims at high productivity and yields of sugar and alcohol, which depend on the amount of water applied, irrigation management combined with the right amount of fertilizer, variety, age of the cut, type of soil and climate (DANTAS NETO *et al*., 2006). In any case, the supply of technological quality raw materials to provide economic extraction is one of the greatest needs of the sugar and alcohol industry (LEITE; CRUSCIOL; SILVA, 2011).

To estimate productivity depending on variations in environmental factors, a function that describes the behavior of these variables can be used. According to Hexem and Heady (1978) and Heady and Dillon (1961), to obtain the production function, ten statistical models proved to be quite satisfactory from field research to represent a production function of a crop.

Therefore, the objective of this work was to identify, through the production function, the doses of N and the irrigation depths that allow for greater productivity and higher yields of sugar and alcohol in sugarcane in 4th cut plants. on soil without and with subsoiling to use resources in the most efficient way possible.

4 MATERIALS AND METHODS

4.1 Experiment setup

The experiment was carried out in the field at the Carpina Sugarcane Experimental Station - EECAC/UFRPE, with coordinates 7° 51' 13" S, 35° 14' 10" W, at 180 m altitude, in ratoon cane on the 4th cultivation sheet, continuing the work started by Costa *et al.* (2016) .

The soil in the experimental area is classified as dystrocohesive Yellow Argisol, according to Embrapa (2013). Chemical (Table 1) and physical (Table 2) characterization of the soil was carried out at depths of 0-20 and 20-40 cm according to Teixeira *et al*. (2017). The area's fertilization history is shown in Table 3.

Table 1. Chemical characterization of the soil in the experimental area.

Chemical analysis 4th cut										
Litter	pH	K^{+}	Ca^{+2}	Mg^{+2}	At $^{+3}$	H+Al	CTC	V	M	Q
cm	H_2O _ _	------------------- cmolc dm $^{-3}$ -----------------						--------%------		mg dm $^{-3}$
0-20	5.1	0.08	1.5	0.3	0.3	3.9	5.78	32.52	13.76	7
20-40	5.0	0.06	0.9	0.3	0.6	4.0	5.26	23.95	32.26	7

pH – Hydrogenion potential in water; K^{+} - Potassium; Ca^{2+} - Calcium; Mg^{+3} – Magnesium; Al^{+3} – Aluminum; H+Al – Hydrogen + Aluminum; CTC – Cation exchange capacity; V – Base saturation; m- Aluminum saturation; P – Phosphorus.

Table 2. Physical characterization of the soil in the experimental area.

Physical analyzes 4th cut								
Layer	Ds ^{1}	DS2 –	Sand	Silt	Clay	ϴCC _	ϴ $_{PMP}$	Textural class
(cm)	Mg m $^{-3}$	Mg m $^{-3}$	---------g kg $^{-1}$ --------			--m $^{-3}$ m $^{-3}$ --		
0-20	1.71	1.54	848.7	13.9	137.4	0.15	0.10	Sandy loam
20-40	1.86	1.55	826.2	16.4	157.4	0.18	0.12	Sandy loam

1 – Soil density without subsoiling; 2- Soil density with subsoiling; ϴ $_{CC}$ – Humidity at field capacity; ϴ $_{PMP}$ – Moisture at the permanent wilting point.

Table 3. Fertilization management during cultivation of the area.

Sheet	P_2O_5 [a]	K_2O [b]	N0	N1	N2	N3	N4	Source
	kg ha $^{-1}$							
1st	30	60	0	20	40	80	120	Urea
2nd	0	80	80	80	80	80	80	Urea
3rd	0	80	0	40	80	120	160	$(NH_4)_2SO_4$
4th*	0	80	0	40	80	120	160	$(NH_4)_2SO_4$

P_2O_5 – Phosphorus oxide; K_2O – Potassium oxide; KCl - Potassium chloride; N – Nitrogen; $(NH_4)_2SO_4$ – Ammonium sulfate; a – Simple super phosphate as source; b- Potassium chloride as source; *The application of N was divided into two doses for doses ≥80 kg ha $^{-1}$, the first when the 3rd leaf was cut and the other 30 days later.

4.2 Treatments and experimental design

The treatments consisted of the application of four irrigation depths (L) and five doses of nitrogen (N) in soil with and without subsoiling in a block design, resulting in a 5×4×2 experiment with 3 replications. The sugarcane variety used in the experiment was RB92579.

Nitrogen doses were N1 = 0; N2= 40; N3 = 80, standard dose according to Cavalcanti (2008); N4 = 120 and N5 =160 kg ha $^{-1}$. The total water depths applied via irrigation were determined according to the crop evapotranspiration (ETC), being (L1, L2, L3 and L4) 0, 50, 100 and 150% of the ETC, respectively, according to Table 4. An ETC of 0% corresponds to planting under rainfed conditions.

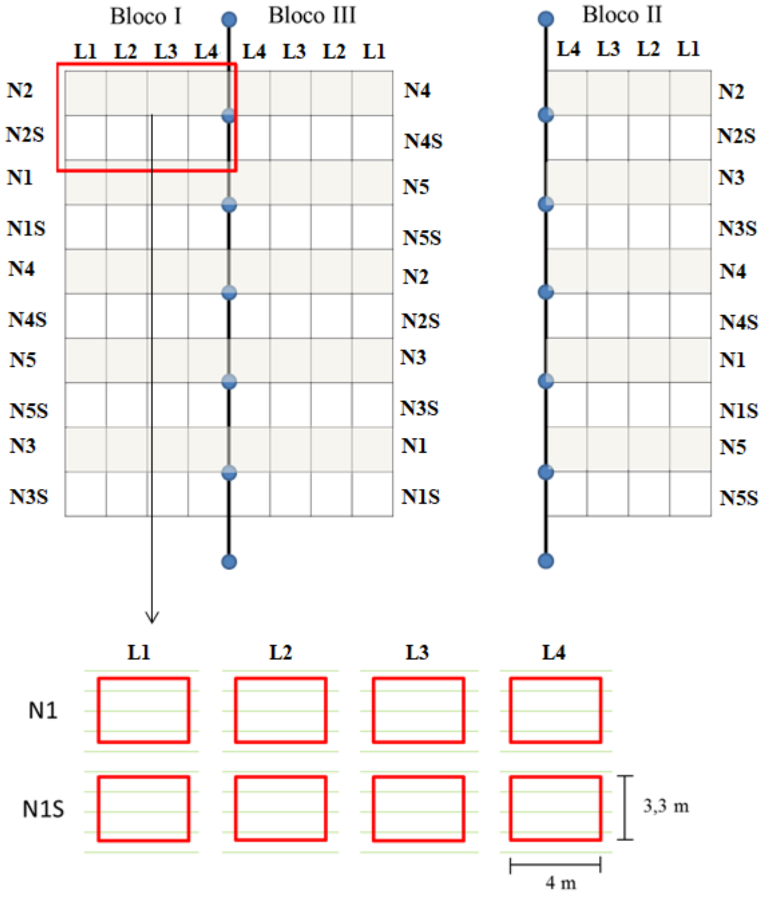
Table 4. Irrigation depths and totals applied in the experiment.

Blade	ETC	Precipitation	Blade applied	Total Blade
	%	mm	mm	mm
L1	0	905	0	905
L2	50	905	275	1180
L3	100	905	551	1456
L4	150	905	826	1731

The experimental plots consisted of five rows of sugarcane plants spaced 1.10 m apart and 6.0 m long, totaling an area of 33.0 ^{m2}. The usable area corresponded to the three central rows minus one meter on each side lengthwise, resulting in a length of 4.0 m and a usable area of 13.2 ^{m2}.

A sketch of the experimental area is shown in Figure 1.

Figure 1. Sketch of the experimental area with details of the experimental plot and treatments. N1, N2, N3, N4 and N5 (0, 40, 80, 120, and 160 kg ha $^{-1,}$ respectively). L1, L2, L3 and L4 (0, 50, 100 and 150% of ETC). OS in front of the factor name indicates that the soil was subsoiled, and the absence of S indicates the soil without subsoiling.



An in-line sprinkler irrigation system was used, namely, the “line source sprinkler system”. The in-line sprinkler irrigation system described by Hanks *et al*. (1976) consists of a single line of overlapping field sprinklers. The arrangement of the sprinklers very close together in a single line allows the overlapping of water jets, providing greater precipitation along the lateral line and a decreasing gradient perpendicular to the pipe. This system, developed for experimental purposes, allows the application of different water depths.

The system consisted of a central line with seven sprinklers spaced every 15 m on a pipe located in the center of the experimental area. The sprinklers used were the KS 1500 mm mini-cannon type, working pressure of 25 mca, nominal flow of 13.61 $m^3 h^{-1}$ and 60 m wet diameter.

At the beginning of the 3rd cultivation cycle, half of each experimental plot was subsoiled to an average depth of 50 cm, thus creating the subsoiled and nonsubsoiled conditions that persisted during the 4th cultivation cycle.

4.3 Irrigation management

The daily evapotranspiration of the crop (ETC - mm) was calculated using data collected in a class A tank located close to the cultivation area using the following equation:

$$ETC=ECA \times Kp \times Kc \qquad (1)$$

Where:
ECA is Class A tank evaporation, mm;
Kp is the Class A tank coefficient;
Kc is the crop coefficient.

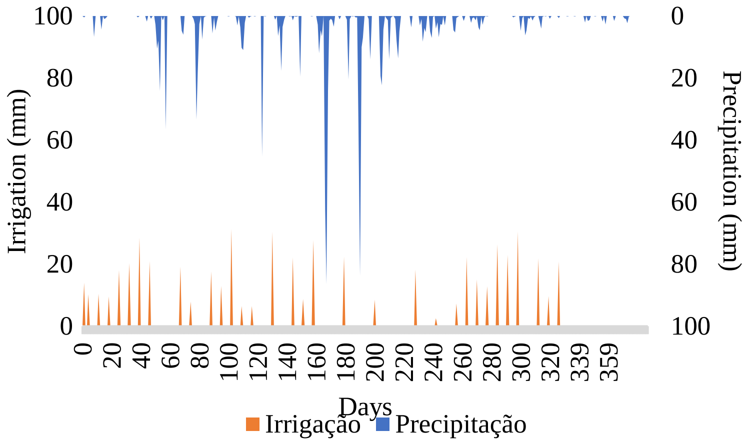
The Kp value was obtained from data from a meteorological station and a class A tank using the values recommended by Doorenbos and Kassam (1994) (Table 5).

Table 5. Kc values for ratoon sugarcane.

Age of Sugarcane (Months)	Development period	Kc
0-1	From planting to 25% coverage	0.55
1-2	From 25 to 50% coverage	0.80
2-2.5	From 50 to 75% coverage	0.90
2.5-4	From 75% to complete coverage	1.00
4-10	Full ground coverage	1.05
10-11	Beginning of Maturation	0.80
11-12	Maturation	0.60

Figure 2 presents precipitation and irrigation depth data during the period.

Figure 2. Precipitation and irrigation depth applied during the experiment.



4.4 Sugar and alcohol yield

The yields of standard sugar and anhydrous alcohol were estimated using the stoichiometric yields of the processes that result in these two products as a function of ART (total reducing sugar). Following the following relationships according to Fernandes (2011):

1.0000 Standard sugar = 1.0526 kg of ART
1.0000 Anhydrous alcohol = 1.7160 kg of ART
1.0000 hydrated alcohol = 1.64736 kg of ART

4.5 Production function

The production factors water (L) and nitrogen (N) constituted the independent variables, and crop productivity (Y) constituted the dependent variable. To obtain the production function, ten statistical models were tested, which, according to Hexem and Heady (1978) and Heady and Dillon (1961), proved to be quite satisfactory from field research to represent a production function of a culture.

Among these models, the one that best fit the experimental data was chosen, taking into account the coefficients of determination R^2, the value of the F test of the analysis of variance for all coefficients, the standard error and the signs of the variables of the models analyzed. The statistical models tested were as follows:

Z=X0+X1.L+X2.N+X3.L^0.5+X4.N^0.5+X5.N^0.5.L^0.5 (2)

Z=X0+X1.L+X2.N+X3.L^0.5+X4.N^0.5+X5.NL (3)

Z=X0+X1.L+X2.N+X3.L^0.5+X4.N^0.5 (4)

Z=X0+X1.L+X2.N+X3.L^2+X4.N^2+X5.LN (5)

Z=X0+X1.L+X2.N+X3.L^2+X4.N^2 (6)

Z=X0+X1.L+X2.N+X3.L^1.5+X4.N^1.5+X5.LN (7)

Z=X0+X1.L+X2.N-X3.L^1.5-X4.N^1.5 (8)

Z=X0+X1.L-X2.L^2-X3.N^2 (9)

Z=X1.L+X2.N-X3.L^2-X4.N^2+X5.LN (10)

Z=X1.L+X2.N-X3.L^2-X4.N^2 (11)

4.6 Region of maximum production

The values of the independent variables, L and N, that maximize production were obtained by deriving the equation that best fit the experimental data in relation to each factor studied and setting it equal to zero (Equations 12 and 13).

$$\partial Z/\partial L=0 \quad (12)$$

$$\partial Z/\partial N=0 \quad (13)$$

where:
$\partial Z/\partial L$= derivative of the function (Z) in relation to the considered factor (L);
$\partial Z/\partial N$ = derivative of the function (Z) in relation to the considered factor (N);

4.7 Marginal rate of technical substitution

The marginal rate of substitution (TMS) that corresponds to the quantity of the blade factor, which can be replaced by the nitrogen factor, maintaining the same level of productivity, was obtained by the relationship between the first derivative of the production function in relation to the blade factor and the first derivative of the production function in relation to the nitrogen factor (Equation 14).

$$TMS=-(\partial Z/\partial N)/(\partial Z/\partial L) \quad (14)$$

where:
$\partial Z/\partial L$= derivative of the function (Z) in relation to the considered factor (L);

$\partial Z/\partial N$ = derivative of the function (Z) in relation to the considered factor (N);

4.8 Isoquants and rational production region

From the production function, the isoquants were determined, plotting the water depth data and nitrogen doses as a function of the previously fixed productivities in a two-dimensional graph. The point on each isoquant where the marginal rate of substitution is zero or infinite delimits the region of rational production.

4.9 Statistical analysis

For the yield of standard sugar, anhydrous alcohol and hydrated alcohol, the F test was performed, and the Tukey test was applied for significant interactions ($P<0.05$) with Sisvar Software (FERREIRA, 1998).

For the parameters of the models that describe the production functions, the F test, calculation of R^2, calculation of the standard error and analysis of the signs of the functions were carried out. Statistical analysis was performed using Statistica 10 Software (STATSOFT, 2011).

5 RESULTS AND DISCUSSION

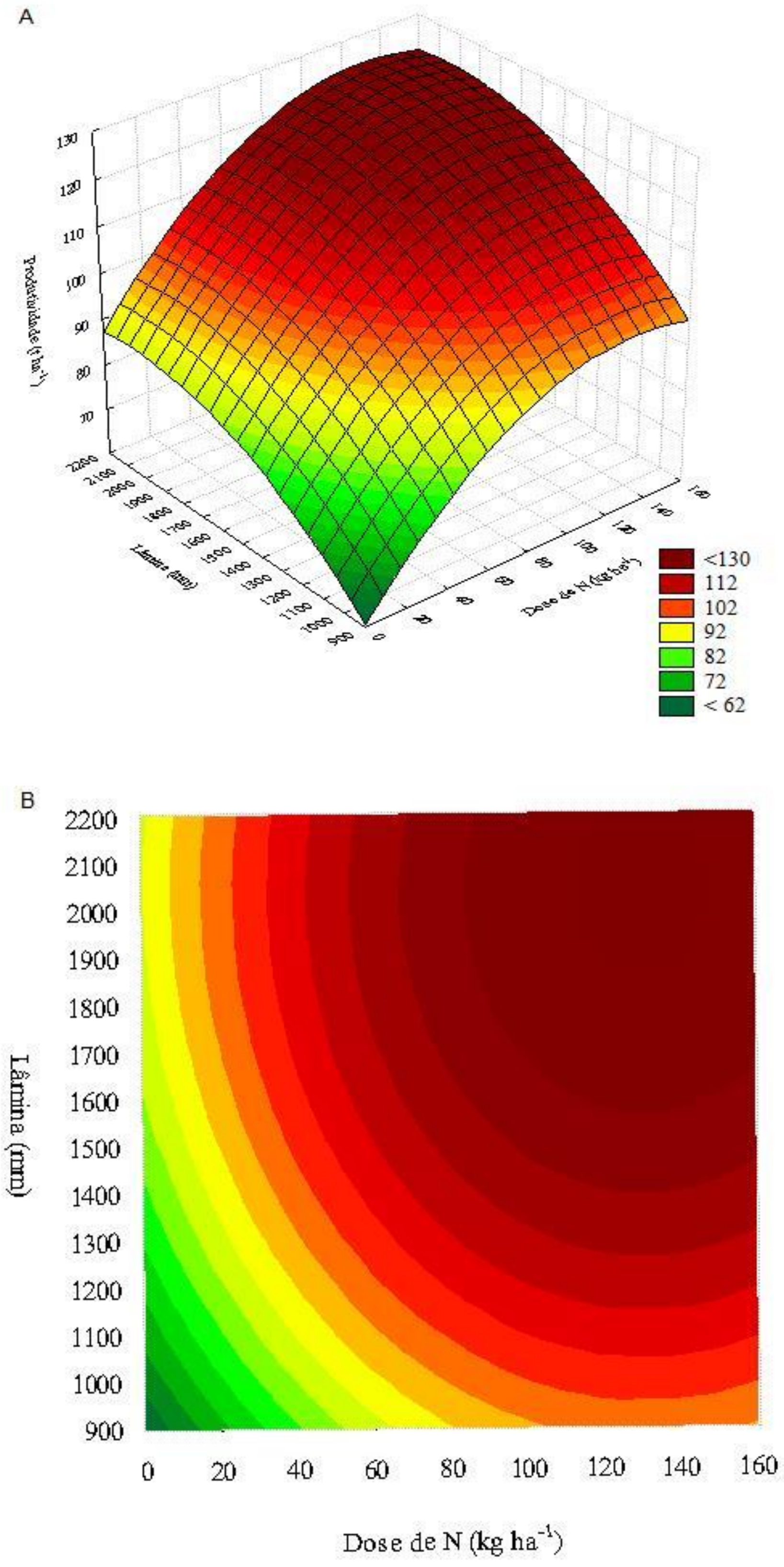
5.1 Production function

After analyzing the parameters of the 10 models that describe the behavior of the production function, only models 10 and 11 presented all significant parameters using the F test ($P<0.01$), with a coefficient of determination (R^2) of 0.69, which guarantees good adjustment and explanation of the variance in soils with and without subsoiling. To represent the behavior of productivity in both cases, equation 11 was chosen, as it also has the smallest standard error in both cases.

The production function describes the behavior of productivity as a function of the N dose (kg ha $^{-1}$) and the total depth (precipitation + irrigation) in sugarcane cultivation. In Figure 3, the behavior of the production function for nonsubsoiled soil is observed, as described in equation 15.

$$TCH = 0.086411L+0.562707N-0.000021L^2-0.002148N^2 \qquad (15)$$

Figure 3. Sugarcane production function as a function of water depths and nitrogen doses in nonsubsoiled soil.



In Figure 3A, it is observed that as the dose of N and the total depth increase, productivity also increases until reaching a maximum point. According to Sigh nand Brar (2015), sugarcane responds well to irrigation, which provides increased productivity.

The isoquants that describe the relationship between the lamina and the N dose can be seen in Figure 3B. Isoquants are curves where the dependent factor, in this case productivity, remains constant while the other independent factors vary. Using these isoquants, it is possible to measure the relationship between N doses and total laminae to achieve the desired productivity.

The maximum production for the ratoon sugarcane production function as a function of water depths and nitrogen doses in the nonsubsoiled soil was obtained by setting the partial derivatives of the function equal to 0, according to equations 16 and 17.

$$\partial Z/\partial N = 0.562707 - 4.296 \times 10^{-3} N = 0 \quad (16)$$

$$\partial Z/\partial L = 0.086411 - 4.2 \times 10^{-5} L = 0 \quad (17)$$

In this way, the maximum productivity found was 125 Mg ha $^{-1}$, obtained from the combination of the 2033 mm blade with a dose of 131 kg ha $^{-1}$ of N. The total maximum productivity blade was 40% higher than that of the 100 mm blade. % ETC (1456 mm), considered standard. For the N dose, the increase in relation to the standard dose (80 kg ha $^{-1}$) exceeded 60%.

Analyzing equation (15), which describes the production function, it can be seen that the treatment that received a dose of 80 kg of N ha $^{-1}$ (standard dose) combined with a level of 100% ETC (N3L3) resulted in a productivity of 112 Mg ha $^{-1}$. The maximum productivity achieved exceeds this value by 12%.

The lowest productivity value achieved by combining the blade (0% ETC – 905 mm) and dose (0 kg ha $^{-1}$) was 64 Mg ha $^{-1}$. Obtaining a 95% increase with maximum productivity. According to Silva, Cato and Costa (2010), an economically productive sugarcane field must be conducted through at least five to six cuts or until the average productivity is close to 65 Mg ha $^{-1}$. The increase in irrigation in the rainfed areas of Brazilian sugarcane fields has substantial potential in increasing biomass (OLIVEIRA *et al.*, 2021).

The marginal substitution rate (TMS) for nonsubsoiled soil is presented in Table 6. Using the TMS, it can be seen how much of the surface area (mm) can be replaced per kg of N while maintaining the same productivity.

Table 6. Values of the marginal rate of substitution (TMS) of water (L) for nitrogen (N) in sugarcane isoquants of 70, 80, 90, 100, 110, 120 Mg ha -1 in nonsubsoil.

	70 Mg ha^{-1}		**80 Mg ha^{-1}**		**90 Mg ha^{-1}**		**100 Mg ha^{-1}**		**110 Mg ha^{-1}**		**120 Mg ha^{-1}**	
N (kg ha^{-1})	L (mm)		L (mm)		L (mm)		L (mm)		L (mm)		L (mm)	
10	986	-11,7	1241	-15,5	1639	-31,1						
20			1107	-12,1	1412	-18,1						
30			997	-9,8	1256	-13,1	1672	-28,7				
40					1140	-10,3	1461	-16,1				
50					1049	-8,3	1327	-11,6	1857	-46,4		
60					974	-6,8	1225	-8,9	1606	-16,8		
70							1147	-6,9	1470	-10,9		
80							1086	-5,4	1380	-7,9		
90							1038	-4,2	1310	-5,7	1810	-18
100							1001	-3,0	1261	-4,1	1679	-8,8
110							975	-2,0	1230	-2,6	1615	-5,1
120							960	-1,0	1209	-1,3	1575	-2,4
130							955	-0,1	1202	-0,1	1561	-0,2
140							960	0,8	1208	1,1	1570	2,0
150							972	1,8	1225	2,9	1605	4,5
160							995	2,8	1255	3,8	1666	8,0

A negative TMS indicates that water is being replaced by nitrogen in decreasing proportions. Obtaining a positive TMS shows that water is being replaced by nitrogen in increasing quantities, making replacement uneconomical.

The analysis of Table 6 allows for a series of combinations of scenarios to obtain the most diverse productivity, ensuring adequate management of N levels and doses. TMS reinforces the importance of adequate management of irrigation and nitrogen fertilization for sugarcane. sugar. According to Vale *et al.* (2013), inadequate management of nitrogen fertilization in sugarcane fields can lead to a reduction in crop yield and longevity, thus reducing the number of cuts between reforms.

Observing the productivity of 110 Mg ha $^{-1}$, the closest to that obtained in N3L3 (80 kg of N ha $^{-1}$ and depth of 1456 mm), it is observed that the lowest dose of N applied to obtain this productivity is 50 kg combined with an 1810 mm blade, with a TMS of -46.53; that is, to maintain the same productivity, the increase of one kg of N is followed by a decrease of 46.53 mm.

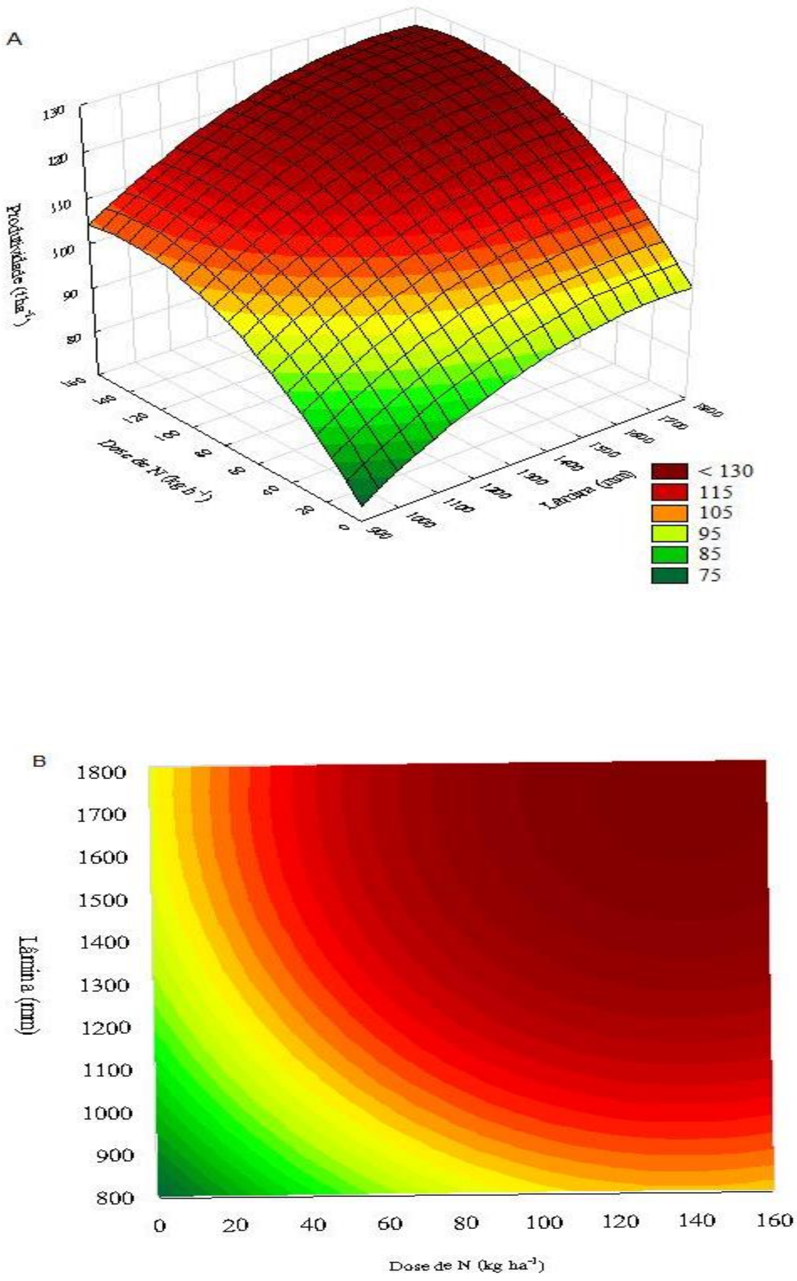
In this same productivity of 110 Mg ha $^{-1}$ above the N dose of 140 kg ha $^{-1,}$ it is uneconomical to replace the blade factor (mm) by kg of N, as from then on, the TMS is positive. The TMS of 1.11 indicates that for each kg of N added, 1.11 mm of water is also added to maintain the same productivity. According to Leite *et al.* (2013), the water factor can be replaced by the nitrogen factor up to a certain point, allowing us to obtain the same productivity after which the volume of water applied exceeds the rational production area.

In Figure 3B, when observing the isoquants, the region of rational production of the sugarcane crop for this study is identified. Given the chosen production function, the water depths and N doses, which delimit the rational production region, correspond to the same values that combined result in maximum production. In this region, the TMS is zero or infinite. The N depth and dose are 2033 mm and 131 kg ha $^{-1}$, respectively.

In Figure 4, the behavior of the production function for subsoiled soil is observed, as described in equation 18.

$$TCH\ sub = 0.110317L+0.458228N-0.000032L^2-0.001661N^2 \tag{18}$$

Figure 4. Sugarcane production function as a function of water depths and nitrogen doses in the subsoil soil.



The maximum production for the ratoon sugarcane production function as a function of water depths and nitrogen doses in the nonsubsoiled soil was obtained by setting the partial derivatives of the function equal to 0 according to equations 1, 9 and 20.

$$\partial Z/\partial N= 0.458228-3.322*10^{(-3)} N=0 \qquad (19)$$

$$\partial Z/\partial L = 0.110317-6.4*10^{(-5)} L=0 \qquad (20)$$

Analyzing the treatment that received a dose of 80 kg of N ha $^{-1}$ (standard dose) with a level of 100% ETC (N3L3), there was a productivity of 118 Mg ha $^{-1}$. The maximum productivity achieved was only approximately 8% higher than the productivity with standard doses. The irrigation depth found was 18% higher than the 100% ETC depth, while for the N dose, the increase recorded was 71%.

In both soil conditions, the productivity values for N3L3 were close to the maximum productivity. According to Roberts (2008), high levels with high doses of N have great potential for losses, mainly through leaching. Mendonça *et al.* (2018), cultivating sugarcane in Minas Gerais, also found high productivity with the application of 80 kg ha $^{-1}$ supplying the crop's ETC.

The lowest productivity value of 74 Mg ha $^{-1}$ was obtained in the blade (0% ETC) and dose (0 kg ha $^{-1}$). Maximum productivity represents an increase of approximately 70% in relation to the lowest productivity found. This value is higher than the 59.73 Mg ha $^{-1}$ found by Costa *et al.* (2016) in sugarcane for the same dose of N and application of 0% ETC. In Figure 3B, the values that combined result in maximum productivity also delimit the region of rational production equally in the nonsubsoiled soil.

Table 7 shows the TMS for subsoiled soil.

Table 7. Values of the marginal substitution rate (TMS) of water (L) for nitrogen (N) in sugarcane cultivation isoquants of 80, 90, 100, 110, and 120 Mg ha $^{-1}$ in subsoil soil.

	80 Mg ha $^{-1}$		**90 Mg ha $^{-1}$**		**100 Mg ha $^{-1}$**		**110 Mg ha $^{-1}$**		**120 Mg ha $^{-1}$**	
N (kg ha $^{-1}$)	L (mm)		L (mm)		L (mm)		L (mm)		L (mm)	
10	1030	-10.3	1170	-11.9	1639	-74,3				
20			1070	-9,3	1403	-18,8				
30			980	-7,5	1256	-11,9				
40					1150	-8,8	1550	-28,5		
50					1060	-6,8	1349	-12,1		
60					995	-5,5	1249	-8,5		
70					951	-4,5	1200	-6,7		
80							1140	-5,1	1456	-13,2
90							1080	-3,8	1400	-7,6
100							1050	-2,9	1350	-5,2
110							1020	-2,05	1290	-3,31
120							1010	-1,30	1270	-2,03
130							1000	-0,57	1250	0,86
140							1000	0,15	1250	0.23
150							1000	0.86	1260	1.34
160							1010	1.6	1280	2.56

Analyzing the productivity of 120 Mg ha $^{-1}$, the closest to that obtained in N3L3 (80 kg of N ha $^{-1}$ and depth of 1456 mm), it is observed that the lowest dose of N applied to obtain this productivity is the same as that used in the N3L3 treatment. Below 80 and above 120 Mg ha $^{-1,}$ the TMS becomes positive, indicating increasing doses of both resources to increase or maintain productivity.

Comparing the soil without and with subsoiling, it is observed that the productivity of 125 Mg ha $^{-1}$ obtained in the soil without subsoiling and 127 Mg ha $^{-1}$ obtained in the soil with subsoiling are quite close, which occurs when the doses are compared. of N; 131 and 137 kg ha $^{-1}$ for soils without and with subsoiling, respectively. When comparing the blades that combined with the dose of N to achieve this productivity, this difference increases significantly. Almost 18% between the applied blades, a difference of 304 mm. These values are lower than the productivity obtained by Costa *et al.* (2016) in sugarcane plants (170 Mg ha $^{-1}$).

The nonsubsoiled soil condition resulted in a greater depth to achieve maximum productivity. According to Alameda *et al.* (2012), soil compaction causes less root development, which makes it difficult for plants to absorb water and nutrients.

According to Cherubin *et al.* (2016), increased soil compaction promotes a decrease in yield potential in the fourth and fifth ratoon. Therefore, it is essential to use practices that reduce soil compaction, such as subsoiling, and minimize damage to the soil structure, which impairs the absorption of water and nutrients and promotes a reduction in sugarcane yield (SOUZA *et al.*, 2014).

Factors such as genotype, plant age, and cultivation region in addition to those studied can influence sugarcane productivity, which is why various productivity values are found in the literature.

Cardozo, Bordonal and Scala Júnior (2016) reported that the average productivity of irrigated sugarcane in Brazil can reach values greater than 140 Mg ha $^{-1}$. Silveira, Barbosa and Oliveira (2002) found yields between 122 and 151 Mg ha $^{-1}$ with the application of conventional fertilizers under irrigation. Dias e Sentelhas (2019) e Silva *et al.* 2019) já demonstraram a viabilidade técnica de irrigação com esta cultura, resultando em consideráveis aumentos de produção. Similar to the values found in this study under irrigation conditions, these values are higher than the average productivity in Brazil, which is 72.61 kg ha $^{-1}$ according to Conab (CANA-DE-AÇÚCAR, 2022), resulting in the understanding that irrigation and fertilization of sugarcane efficiently provide greater productivity for the producer.

5.2 Sugar and alcohol yield

According to Moraes *et al.* (2016), it is important to know and understand the correlations between the productive and technological attributes of the plant and those of the soil to contribute not only to the reduction of costs but also as a way to increase the productivity of the plant. culture and the quality of the harvested material.

Understanding the interaction between the factors that limit productivity is extremely important for the sugar and alcohol industry, as this allows us to identify the appropriate management to achieve the highest yields.

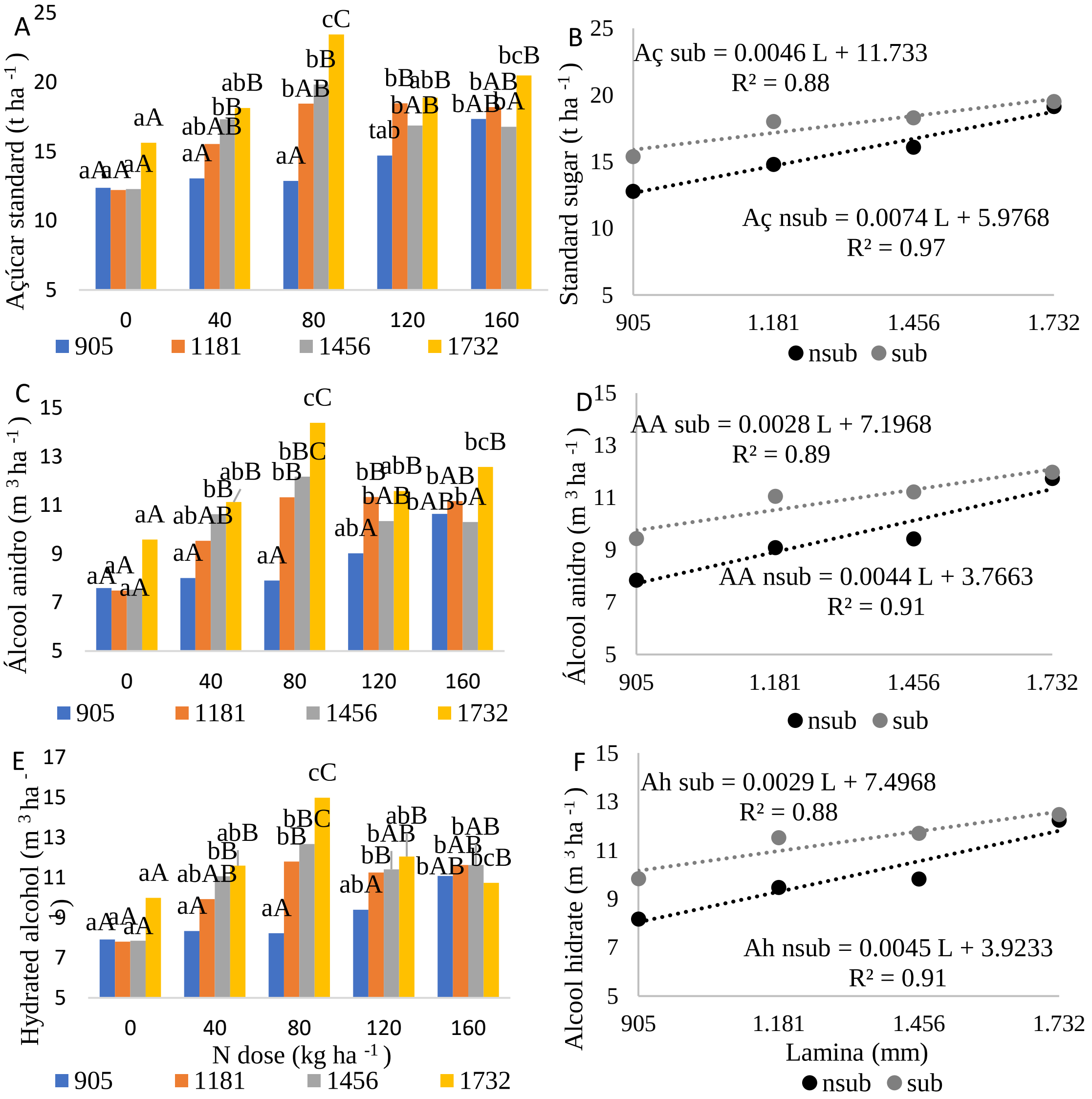
Table 8 presents the analysis of variance for the yield of sugar (Aç), anhydrous alcohol (AA) and hydrated alcohol (Ah).

Table 8. Summary of the ANOVA of sugar and alcohol yield from sugarcane as a function of water depth (L) and nitrogen dose (N).

Fv	GL	QM AÇ	QM AA	QM AH	F
Block	two	5.58	2.10	2.28	0.36ns
N	4	118.24	44.49	48.27	7.70**
error 1	8	15.34	5.77	6.26	
L	3	139.58	52.52	56.99	6.43*
error 2	6	21.70	8.16	8.86	
Sub	1	155.10	58.36	63.32	206.14**
error 3	two	0.75	0.28	0.30	
NxL	12	15.34	5.77	6.26	2.93*
NxS	4	13.46	5.07	5.50	2.57ns
LxS	3	16.02	6.03	6.54	3.06*
NxWxS	12	90.06	3.41	3.70	1.73ns
error 4	23	5.22	1.97	2.13	
Total	119				

Figure 5 presents the analysis of the interactions between the N doses and the blades and between the blades and the soil without and with subsoiling.

Figure 5. Breakdown of the interaction between N doses and the layers (A, C and E) and between the layers and the nonsubsoiled (nsub) and subsoiled (sub) soil (B, D, F) for sugar yield standard (Aç), anhydrous alcohol (AA) and hydrated alcohol (Ah).



Lowercase letters for the breakdown of the slides within each dose and capital letters for the breakdown of the doses within each slide.

In Figure 5A, 5C and 5E, the breakdown of the N doses within each slide and the slides within each N dose for standard sugar, anhydrous alcohol and hydrated alcohol, respectively, can be seen.

The highest value found for the yield of sugar, anhydrous alcohol and hydrated alcohol occurred in the interaction between the 1732 mm depth and the dose of 80 kg of N ha $^{-1}$. With the exception of the 0 kg ha $^{-1}$ dose, the sugar yield was lower at 1732 mm.

The 905 mm depth in all doses also showed low yields that only differed in the dose of 160 kg of N ha $^{-1}$. The lack of water was compensated by a greater supply of N.

The smallest blade was not enough to meet all the plant's needs, which caused a drop in yield. Water deficiency is not limited to arid and semiarid regions of the world; in regions considered humid, the irregular distribution of rainfall can, in some periods, limit development and harm crop productivity (SILVA., 2014 *et al*).

Water stress affects plants in several ways. The plant closes its stomata to reduce water loss and ends up absorbing less CO2 that would be used in photosynthesis, causing a decrease in net photosynthesis. In addition to the effects on the biochemical phase of photosynthesis, there is damage to the photosystems responsible for capturing the light photon, which also reduces the performance of the photochemical phase.

The lack of water also hinders the absorption of nutrients, and these combined effects result in a decrease in productivity and yield of sugar and alcohol. According to Bastos *et al*. (2016), water interacts with N to influence the total recoverable sugar.

Lack of N also plays a role in the lower yields found. NO participates in the composition of several molecules in plants, and the low availability of this nutrient causes a decrease in growth, leaf area and lower levels of photosynthetic pigments. According to Rhein *et al*. (2016), sugar productivity and yield increase linearly with the increase in nitrogen doses applied.

In Figure 5B, the breakdown of the N dose for soils without and with subsoiling for standard sugar yield can be seen. The higher the irrigation depth was, the higher the sugar yield. Furthermore, subsoiled soil provided greater yields for the crop compared to nonsubsoiled soil. An average increase of 13% in sugar yield was observed.

Applying the maximum depths in the equations that describe the behavior of sugar yield, we find 19.7 and 18.79 t of sugar ha $^{-1}$ for the soil with and without subsoiling, respectively. This represents an increase of approximately 5% in yields for subsoiled soil. Comparing the sugar yields for 1456 mm (100% of ETC), this difference increases to approximately 10% with 18.43 and 16.75 Mg ha $^{-1}$ for the soil with and without subsoiling.

The breakdown of anhydrous alcohol yield as a function of the total depth in the soil with and without subsoiling can be seen in Figure 5D. The subsoiled soil at maximum depth presented a yield of 12.04 m³ ha $^{-1,}$ while in the nonsubsoiled soil, the yield was 11.37 m³ ha $^{-1}$, a difference of 6%.

For hydrated alcohol (Figure 5F), the yields at maximum depths were 11.71 m ^{3} ha $^{-1}$ and 13.07 m ^{3} ha $^{-1}$, resulting in an approximately 12% difference between the soils. Notably, these differences change as the blade applied decreases, reaching an average difference of 15% in both cases.

6 CONCLUSIONS

Subsoiling the soil provided less water application compared to nonsubsoiled soil to obtain similar productivity.

N doses between 80 and 160 kg ha $^{-1}$ and total depths between 1456 and 1731 mm provided the highest yields of sugar and alcohol.

The combinations of nitrogen doses and water depths chosen to achieve higher sugar and alcohol productivity must be located within the rational production region.

Function 11 where Z=X1.L+X2.N-X3.L^2-X4. N^2 was the one that best fit the experimental data.

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