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MODELO MATEMÁTICO APLICADO À FERTIRRIGAÇÃO[1]

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1 RESUMO

O objetivo deste trabalho foi propor um modelo matemático que auxilie na determinação de uma mistura ótima de fertilizantes visando a minimização do custo da solução nutritiva. Dessa maneira, foi desenvolvida uma metodologia que permitisse identificar uma mistura ótima de fertilizantes capaz de auxiliar o produtor no manejo da fertirrigação. Essa metodologia consistiu no desenvolvimento de um modelo matemático de otimização para auxílio na determinação da quantidade ótima de fertilizante a ser inserida na mistura de forma a suprir as necessidades nutricionais da cultura a um custo mínimo. Para auxiliar o usuário desse modelo, foi desenvolvido um aplicativo para dispositivos móveis. Sendo, portanto, esse aplicativo uma ferramenta de apoio aos produtores no processo de tomada de decisões à campo. Dentre os métodos de resolução de problemas de programação não-linear, optou-se pelo Método do Gradiente Conjugado Não-Linear; sendo esse, um caso particular do Método do Gradiente Conjugado, proposto para soluções de problemas não-lineares. O modelo matemático proposto atendeu às restrições preconizados na fertirrigação para o cálculo da quantidade de fertilizantes, à um baixo custo, tornando-o mais completo e eficiente. Dessa maneira, a metodologia proposta e o aplicativo desenvolvido são ferramentas importantes e acessíveis que podem auxiliar os produtores no manejo da fertirrigação.

Palavras-chave: adubação, condutividade elétrica, custo, modelagem, programação não-linear.

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MATHEMATICAL MODEL APPLIED TO FERTIGATION

2 ABSTRACT

The objective of this work was to develop a mathematical model to assist farms in the determination of an optimal mixture of fertilizers in order to minimize the cost of nutritive solutions. This way, we developed a methodology that allows to identify an optimal mixture of fertilizers capable to assist farms in the management of fertigation. The methodology consisted of the development of an optimization mathematical model to assist in the determination of the optimal amount of fertilizers to be insert in the mixture in order to meet the nutritional needs of

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the crop at a minimum cost. In order to assist the user of this model, a mobile application was developed. Thus, this mobile application is a decision support tool to farms. Among the nonlinear programming methods, we opted for the Nonlinear Conjugate Gradient Method, this method is a particular case of the Conjugate Gradient Method, proposed to provide solutions of nonlinear problems. The proposed mathematical model complied with the restrictions recommended in fertigation regarding the calculation of the amount of fertilizer at a low cost; making it more efficient and complete. Therefore, the proposed methodology and the developed mobile application are important and accessible tools that could assist farms in the management of fertigation.

Keywords: fertilization, electrical conductivity, cost, modeling, nonlinear programming.

3 INTRODUCTION

With significant population growth, it is essential to use land and water resources efficiently. According to the World Health Organization (WHO) (2019), the global population is estimated to reach 9.7 billion by 2050, representing an increase of 2.3 billion people. This growth indicates that food demand could be 60% higher than the current level (FAO, 2001).

In this context, irrigated agriculture is essential for many countries, as it can promote food security and the development or increase of agricultural areas, especially in arid and semiarid regions, where rainfall is scarce or irregular throughout the year (SHAMIR et al., 2015).

In recent years, with the advancement of information technology and agricultural activities, applied studies on the optimization of agricultural systems have intensified; that is, greater productivity combined with lower production costs has been achieved. In this context, mathematical optimization modeling is a highly useful scientific tool, as it can aid in predicting the variables involved and in decision-making regarding the system under study, thus avoiding the high costs of field research and improving resource utilization (CORRÊA et al., 2011; TEIXEIRA, 2015; TEJADA-CASTRO et al., 2019; YOUSIF et al., 2018).

However, one of the major issues regarding the use of technologies in the field is the development of instruments/tools that allow for efficient management of the irrigation system and are easy to apply.

In situations where decision-making involves the allocation of scarce resources, efficient methods are needed to assist planners in the decision-making process. To solve this type of problem, mathematical optimization models are the most appropriate, as they are capable of optimally quantifying the use of limited resources, given that in certain cases, the costs are not only economic but also environmental, political, and social.

Solving an optimization model depends on the availability of data and the existence of specific computational algorithms for this purpose. In general, there are specific resolution methods that explore the complexity, form of the objective function, existence or absence of constraints, and, if applicable, geometry of the feasible region (YEH, 1985; LUENBERGER; YE, 2008).

Mateus and Luna (1986) noted that to compare algorithms for solving nonlinear programming problems (NPPP), criteria based on computational simplicity, time spent on computational processing until reaching an optimal point, memory required during processing and sensitivity to computational errors can be used.

In this context, an appropriate fertigation model should balance the plant's nutritional demand and the grower's fertilizer application (the quantity-timing ratio) while also reducing leaching and environmental contamination. Owing to the importance of optimizing fertigation, several studies have been conducted to investigate the impact of different strategies on soil nutrient distribution, including tests with water and fertilizer levels, irrigation and fertigation intervals, and injection and concentration times. (COELHO et al., 2018; DELAZARI et al., 2018, ROCHA; CHRISTOFIDIS, 2015).

Sinha, Rao, and Bischof (1999) proposed a nonlinear optimization model for the study of multipurpose water reservoirs, aiming to minimize consumption and costs related to water use. Saad and Frizzone (1996) formulated an NLP model to maximize the net revenue of crops irrigated by localized irrigation systems, which is applicable to regular irrigation areas with uniform slopes and emissions.

Fertigation can improve the yield and quality of agricultural production, optimize its effects and save resources through rational process management, considering both the soil characteristics for nutrient transport and the physiological needs of the plant. Therefore, optimal irrigation planning is important for improving the efficiency of water capture and nutrient application (DONG et al., 2018).

Therefore, this work aimed to propose a mathematical model that helps in determining an optimal mixture of fertilizers aimed at minimizing the cost of the solution, considering the compatibility restrictions between fertilizers, the electrical conductivity of the solution and the fertilization recommendation.

4 MATERIALS AND METHODS

To meet the objective of this work, a methodology was developed that allowed the identification of an optimal mixture of fertilizers capable of helping the producer manage fertigation, making it more effective in relation to the use of different fertilizers, by determining their quantities on the basis of the concept of “spoon-feeding” (VILLAS BOAS; ZANINI; FEITOSA FILHO, 2002), providing the soil with the nutrients required according to each phenological stage of the crop.

This methodology consists of developing a mathematical optimization model to help determine the optimal amount of each fertilizer to be added to the mixture to meet the nutritional needs of the crop at the lowest cost. To assist users of this model, the development of a mobile app was also proposed. This app serves as a tool to support technicians and producers in the decision-making process.

In this context, the physiological characteristics of the analyzed crops and their respective daily nutrient absorption rates were considered. Additionally, the specific chemical composition of each fertilizer was considered, with particular attention given to solubility, the electrical conductivity of the solution, and compatibility.

To formulate the model, the following notations were considered: $i$where the index represents the Ktypes of fertilizers that can be used in fertigation, $i = 1, \dots, K$; $z$the index that represents the Zphenological stages, $z = 1, \dots, Z$; $d$the index referring to the days that make up the phenological stage; $d = 1, \dots, D$;and $f$the index that represents fertigation, $f = 1, \dots, F$.

Since $X_{izdf}$the variable that defines the amount of fertilizer $i$to be used in fertigation $f$on day d, of the phenological stage z, the following model is proposed (Equations (1) to (16)):

$$Min \sum_{i=1}^{K} \sum_{z=1}^{Z} \sum_{d=1}^{D} \sum_{f=1}^{F} C_i X_{izdf} + \sum_{(m,n)\in SR} \sum_{z=1}^{Z} \alpha C_m X_{mzdf} C_n X_{nzdf} \quad (1)$$

Subject to:

$$\sum_{i=1}^{K} N_i X_{izdf} = \bar{N}_{zdf}, z = 1, \dots, Z, d = 1, \dots, D, f = 1, \dots, F \quad (2)$$

$$\sum_{i=1}^{K} P_i X_{izdf} = \bar{P}_{zdf}, z = 1, \dots, Z, d = 1, \dots, D, f = 1, \dots, F \quad (3)$$

$$\sum_{i=1}^{K} K_i X_{izdf} = \bar{K}_{zdf}, z = 1, \dots, Z, d = 1, \dots, D, f = 1, \dots, F \quad (4)$$

$$\sum_{i=1}^{K} Ca_i X_{izdf} = \overline{Ca}_{zdf}, z = 1, \dots, Z, d = 1, \dots, D, f = 1, \dots, F \quad (5)$$

$$\sum_{i=1}^{K} Mg_i X_{izdf} = \overline{Mg}_{zdf}, z = 1, \dots, Z, d = 1, \dots, D, f = 1, \dots, F \quad (6)$$

$$\sum_{i=1}^{K} S_i X_{izdf} = \bar{S}_{zdf}, z = 1, \dots, Z, d = 1, \dots, D, f = 1, \dots, F \quad (7)$$

$$\sum_{i=1}^{K} B_i X_{izdf} = \bar{B}_{zdf}, z = 1, \dots, Z, d = 1, \dots, D, f = 1, \dots, F \quad (8)$$

$$\sum_{i=1}^{K} Zn_i X_{izdf} = \overline{Zn}_{zdf}, z = 1, \dots, Z, d = 1, \dots, D, f = 1, \dots, F \quad (9)$$

$$\sum_{i=1}^{K} Mn_i X_{izdf} = \overline{Mn}_{zdf}, z = 1, \dots, Z, d = 1, \dots, D, f = 1, \dots, F \quad (10)$$

$$\sum_{i=1}^{K} Cu_i X_{izdf} = \overline{Cu}_{zdf}, z = 1, \dots, Z, d = 1, \dots, D, f = 1, \dots, F \quad (11)$$

$$\sum_{i=1}^{K} Fe_i X_{izdf} = \overline{Fe}_{zdf}, z = 1, \dots, Z, d = 1, \dots, D, f = 1, \dots, F \quad (12)$$

$$\sum_{i=1}^{K} Mo_i X_{izdf} = \overline{Mo}_{zdf}, z = 1, \dots, Z, d = 1, \dots, D, f = 1, \dots, F \quad (13)$$

$$\sum_{i=1}^{K} eq_i x_{izdf} \leq 40, z = 1, \dots, Z, d = 1, \dots, D, f = 1, \dots, F \quad (14)$$

$$X_{azdf}X_{bzdf} = 0, \; para\; todo\; par(a,b) \in I, \tag{15}$$

$$X_{izdf} \geq 0, i = 1, \ldots, K, z = 1, \ldots, Z, d = 1, \ldots, D, f = 1, \ldots, F \tag{16}$$

$$I = \{(a,b)\; tal\; que\; o\; fertilizante\; a\; \text{é}\; incompatível\; com\; o\; fertilizante\; b\},$$
$$a \in \{1, \ldots, K\}\; b \in \{1, \ldots, K\}$$

The objective function (1) aims to minimize the total cost of fertilizers to be inserted into the optimal mixture to be applied in each fertigation of a given crop. where C_i(R\$ kg $^{-1}$) is the cost of the fertilizer i and where α ($\alpha = M \gg 0$) is the penalty term applied to the costs of fertilizers m and n, which, when mixed, present reduced solubility, is SR $= \{(m,n)/\, m \in \{1,2,\ldots,k\}\; n \in \{1,2,\ldots,k\}\}$.

In this way, the model penalizes fertilizer pairs with reduced solubility (m,n), making it difficult to select these types of fertilizers simultaneously. The model will only make this selection if truly necessary, as the penalty will increase the total cost of the mixture. X_{izdf} is the variable that indicates the amount of fertilizer $i$to be added to the solution tank (kg ha $^{-1}$) to be used in the day's $d$fertigation f at the phenological stage z.

Mathematical expressions (2)-(16) represent the constraints of the proposed problem, where Equations (2)-(13) ensure that the amount of fertilizer to be added to the mixture meets the fertilization recommendation regarding macronutrients and micronutrients proposed by the producer; constraints (14) refer to the electrical conductivity of the solution; constraints (15) prevent two (a,b) incompatible fertilizers from being used concomitantly; and constraints (16) ensure the nonnegativity of the model's decision variables. Additionally, all indexes, parameters and variables integrated into the model are described in Tables 1 and 2.

Frame 1. Descriptions of the indexes, parameters and variables that make up models (11)-(25).

Caption	Category	Description
K	Parameter	Quantity of types of nutrient sources.
$i = 1, \dots, K$	Index	Indexes representing the types of fertilizers.
Z	Parameter	Number of phenological stages of the crop.
$z = 1, \dots, Z$	Index	Indexes representing phenological stages.
D	Parameter	Number of days corresponding to each phenological stage of the crop.
$d = 1, \dots, D$	Index	Indexes representing the days of the phenological stage.
F	Parameter	Number of fertigations carried out in one day.
$f = 1, \dots, F$	Index	Indexes representing fertigation.
C_i	Parameter	Fertilizer cost$i$ (R$ kg $^{-1}$).
α	Parameter	Penalty applied to the use of reduced solubility fertilizers.
N_i	Parameter	Proportion of nitrogen (N) contained in the fertilizer i(decimal).
$\bar{N}$	Parameter	Recommended amount of nitrogen (N) in fertilization (kg ha $^{-1}$).
P_i	Parameter	Proportion of phosphorus (P) contained in the fertilizer i(decimal).
$\bar{P}$	Parameter	Recommended amount of phosphorus (P) in fertilization (kg ha $^{-1}$).
K_i	Parameter	Proportion of potassium (K) contained in the fertilizer i(decimal).
$\bar{K}$	Parameter	Recommended amount of potassium (K) in fertilization (kg ha $^{-1}$).
Ca_i	Parameter	Proportion of calcium (Ca) contained in the fertilizer i(decimal).
$\overline{Ca}$	Parameter	Recommended amount of calcium (Ca) in fertilization (kg ha $^{-1}$).
Mg_i	Parameter	Proportion of magnesium (Mg) contained in the fertilizer$i$ (decimal).
$\overline{Mg}$	Parameter	Recommended amount of magnesium (Mg) in fertilization (kg ha $^{-1}$).
S_i	Parameter	Proportion of sulfur (S) contained in the fertilizer i(decimal).
$\bar{S}$	Parameter	Recommended amount of sulfur (S) in fertilization (kg ha $^{-1}$).
B_i	Parameter	Proportion of boron (B) contained in the fertilizer i(decimal).
$\bar{B}$	Parameter	Recommended amount of boron (B) in fertilization (g ha $^{-1}$).
Zn_i	Parameter	Proportion of zinc (Zn) contained in the fertilizer i(decimal).
$\overline{Zn}$	Parameter	Recommended amount of zinc (Zn) in fertilization (g ha $^{-1}$).

Frame 2. Description of the indexes, parameters and variables that make up models (11)-(25).

Mn_i	Parameter	Proportion of manganese (Mn) contained in the fertilizer i(decimal).
$\overline{Mn}$	Parameter	Recommended amount of manganese (Mn) in fertilization (g ha $^{-1}$).
Cu_i	Parameter	Proportion of copper (Cu) contained in the fertilizer i(decimal).
$\overline{Cu}$	Parameter	Recommended amount of copper (Cu) in fertilization (g ha $^{-1}$).
Fe_i	Parameter	Proportion of iron (Fe) contained in the fertilizer i(decimal).
$\overline{Fe}$	Parameter	Recommended amount of iron (Fe) in fertilization (g ha $^{-1}$).
Mo_i	Parameter	Proportion of molybdenum (Mo) contained in the fertilizer i(decimal).
$\overline{Mo}$	Parameter	Recommended amount of molybdenum (Mo) in fertilization (g ha $^{-1}$).
$I = \{(a, b)\}$	Parameter	Set of indexes referring to pairs of fertilizers that are incompatible.
$SR = \{(m, n)\}$	Parameter	Set of indexes referring to pairs of fertilizers that have reduced solubility when used together.
eq_i	Parameter	Gram equivalent for fertilizer i.
X_i	Decision variable	Amount of fertilizer $i$to be used in fertigation $f$on the day d, at the phenological stage z.

Note: Chlorine was not considered, as its requirement by plants is very low, being in the range of 100--200 mg kg-1 (Xu et al., 2000).

Model (1)- (16) is a nonlinear optimization problem that was solved via the nonlinear conjugate gradient method (NLGC) and implemented in GNU/Octave software. The NLGC method is a particular case of the conjugate gradient method proposed by Hestenes and Stiefel (1952) for solutions of nonlinear problems and/or the minimization of convex functions.

The computational tests were performed on a Dell Core i7 computer with 8 GB of RAM and Windows 10. To validate this model, computational tests were performed with data collected in the literature. Data from Technical Bulletin 215 of the Agronomic Institute of Campinas (TRANI, 2015) were used to determine the crop fertilization recommendation on the basis of experiments carried out under protected cultivation, in which the fertigation technique was used. Data regarding the prices of commercialized fertilizers were obtained from the National Supply Company (CONAB, 2018).

4 RESULTS AND DISCUSSION

® spreadsheets were used to tabulate the data. The types of fertilizers most commonly used in fertigation were considered; therefore, the fertilizers used in the development of the model were ammonium nitrate, calcium nitrate, sodium nitrate, ammonium sulfate, urea, simple superphosphate, triple superphosphate, phosphoric acid, potassium chloride, potassium sulfate, double sulfate of potassium and magnesium, monoammonium phosphate (MAP), MAP crystals, diammonium phosphate (DAP), urea phosphate, potassium nitrate, potassium saltpeter, potassium phosphite, monopotassium phosphate (MKP), bipotassium phosphate, calcium chloride pentahydrate, calcium chloride, bihydrate, calcium sulfate (gypsum), magnesium nitrate, magnesium sulfate, sulfuric acid, nitric acid, Fe EDTA, Zn EDTA, Cu EDTA, borax, Solubor, boric acid, sodium molybdate, ammonium molybdate, copper sulfate, ferrous sulfate, iron sulfate, ferric chloride, manganese sulfate, zinc sulfate heptahydrate, zinc sulfate monohydrate, and cobalt sulfate. The chemical composition of each fertilizer was tabulated, as were the amounts of nutrients present.

The decimal composition of the nutrients contained in the fertilizers was determined via the atomic weights of the elements and the respective molar masses of the fertilizers. With respect to the data related to the compatibility characteristics between the fertilizers, the compatibility table described by Landis (1989) was used.

To determine the electrical conductivity of the solution, it was necessary to relate the equivalent weights of each fertilizer, which were calculated according to Equation (17) and are presented in Table 1, while accounting for the equivalent quantities of cations and anions generated, according to the methodology presented by Dimenstein (2017).

$$eq = \frac{m}{E} \quad (17)$$

where:

eq= gram-equivalent amount of a specific mass of fertilizer;

m= mass of fertilizer (g);

E= Gram-equivalent fertilizer.

Table 1. Molar mass (M) and equivalent weight (PE) of the most commonly used fertilizers in fertigation.

i	Fertilizer	Equivalent Weight (g)
1	Ammonium nitrate	80.0434
2	Calcium nitrate	82.0439
3	Sodium nitrate	84.9947
4	Ammonium sulfate	66.0698
5	Urea	30.0277
6	Simple superphosphate	203.1118
7	Triple superphosphate	135.0415
8	Phosphoric acid	32.6651
9	Potassium chloride	74.5513
10	Potassium sulfate	87.1296
11	Double sulfate of potassium and magnesium	103.7486
12	Monoammonium phosphate (MAP)	115.0257
13	Crystal MAP	115.0257
14	Diammonium phosphate (DAP)	66.0281
15	Urea phosphate	158.0504
16	Potassium nitrate	101,1032
17	Potassium saltpeter	93.0490
18	Potassium phosphite	60.0431
19	Monopotassium phosphate (MKP)	68.0428
20	Bipotassium phosphate	87.0880
21	Calcium chloride pentahydrate	100.5302
22	Calcium chloride bihydrate	73.5073
23	Calcium sulfate (gypsum)	86.0856
24	Magnesium nitrate	128,2033
25	Magnesium sulfate	171.2686
26	Sulfuric acid	49.0393
27	Nitric acid	63.0128
28	Fe EDTA	172.0280
29	Zn EDTA	176.7955
30	Cu EDTA	183.8898
31	Borax	190.6861
32	Solubor	206.2605
33	Boric acid	20.6110
34	Sodium molybdate	120.9839
35	Ammonium molybdate	205.9996
36	Copper sulfate	124.8425
37	Ferrous sulfate	139.0073
38	Iron sulfate	78.6565
39	Ferric chloride	90.0986
40	Manganese sulfate	111.5309
41	Zinc sulfate heptahydrate	143.7748
42	Zinc sulfate monohydrate	89.7290
43	Cobalt sulfate	140.5514

The proposed mathematical model is subject to constraints on fertilizer recommendations, fertilizer compatibility, and solution electrical conductivity, which express characteristics to be considered in fertigation management. The fertilizer recommendation constraints (Equations 2--13) correspond to the nutritional needs of a crop for macronutrients and micronutrients according to its phenological stage.

Therefore, the sum of the nutrients contained in the fertilizer quantities to be calculated must equal the fertilization recommendation previously entered into the model/application. Electrical conductivity is an important variable in fertigation planning; monitoring it allows for more assertive decision-making during fertilization and management of the fertigated crop, contributing to more economical production.

The electrical conductivity constraints of the solution (Equation (14)) represent the amount of equivalents that a fertilizer mixture can add to the solution. Therefore, the sum of the equivalents contained in the fertilizer quantities to be calculated must be less than or equal to 40 total equivalents. The fertilizer compatibility constraints (Equation (15)) refer to which fertilizers can or cannot be mixed together, thus preventing the addition of a fertilizer that may be incompatible with the solution.

For the computational evaluation of the mathematical model of fertigation (Equations 1--16), data extracted from the literature were used, which was based on the fertilization recommendation for tomato crops in greenhouses (protected environments), present in Bulletin 215 of the Agronomic Institute of Campinas (TRANI, 2015). The amounts of nutrients recommended for tomato fertigation are presented in Table 2.

Table 2. Fertigation [(a)] in quantities of nutrients for tomato cultivation under protected cultivation during different periods of crop development.

Macronutrients						
Period	N	P_2O_5	K_2O	Here	Mg	S
DAT (2)			g[(c)]			
10 to 35 days	1800	2571	3568	428	204	353
36 to 60 days	3478	3086	5238	1850	640	895
61 to 90 days	4065	2237	8427	2238	1299	2311
91 to 150 days	10398	4551	22332	6068	2162	5007
Total	19741	12445	39565	10584	4305	8566
Micronutrients						
Period	B	Ass	Faith	Mn	Mo	Zn
DAT (2)			g[(c)]			
10 to 35 days	6	6	23	6	1	2
36 to 60 days	7	7	26	7	1	3
61 to 90 days	9	9	36	9	2	4
91 to 150 days	18	18	73	18	4	7
Total	40	40	158	40	8	16

(a) Carry out fertigation three times a week, observing the compatibility between the fertilizers used.
(b) Days after transplanting seedlings to an agricultural greenhouse.
(c) Amount of nutrients, in grams, per period, for 1000 tomato plants.

Source: TRANI (2015).

After the model was implemented, the output data obtained were analyzed with reference to the nutrient sources indicated by the Bulletin of the Agronomic Institute of Campinas (TRANI, 2015) (Table 3).

Table 3. Fertigation ([a]) in quantities of fertilizers for tomato plants under protected cultivation (agricultural greenhouse) during different periods of crop development.

Period	**MAP**	**$MgSO_4$**	**K_2SO_4**	**KNO_3**	**$Ca(NO_3)_2$**	**Micro**
DAT[(b)]			g[(c)]			
10 to 35 days	400	200	50	700	200	30
36 to 60 days	500	650	50	1100	900	35
61 to 90 days	300	1100	300	1200	900	40
91 to 150 days	300	900	550	1500	1200	40
Total	**1500**	**2850**	**950**	**4500**	**3200**	**145**

(the) Carry out fertigation three times a week, observing the compatibility between the fertilizers used.
(b) Days after transplanting seedlings to an agricultural greenhouse.
(c) Amount of nutrients, in grams, per period, for 1000 tomato plants.

Source: TRANI (2015).

By applying the model, the cost of the optimal fertilizer mixture was determined, as were the types to be used in fertigation to meet the nutritional demand indicated for the table tomato crop (Table 2). The types of fertilizers indicated by the model were urea, simple superphosphate, potassium chloride, monopotassium phosphate, magnesium nitrate, magnesium sulfate, boric acid, ammonium molybdate, copper sulfate, iron sulfate, manganese sulfate, and zinc sulfate (Table 4).

Table 4. Fertigation ([a]) in quantities of fertilizers and their respective costs for tomato plants under protected cultivation (agricultural greenhouse) in the different periods of crop development on the basis of the application of the model.

i	**Fertilizer**	**Unit price**	**Amount DAT[(b)]** 10 to 35 days	36 to 60 Days	61 to 90 Days	91 to 150 Days
		(R$ kg^{-1})	kg[(c)]			
5	Urea	1,840	0.3411	0.5949	0.6513	0.8564
6	Simple superphosphate	1,800	0.2024	0.9115	0.9126	1,2162
9	Potassium chloride	2,080	0.0946	0.4162	1,0271	1,4740
19	Monopotassium phosphate (MKP)	6,350	0.9865	1.0128	0.4851	0.3833
24	Magnesium nitrate	3,140	0.0812	0.5551	0.2130	0.1073
25	Magnesium sulfate	2,080	0.1598	0.1353	1,1885	1.0617
33	Boric acid	4,200	0.0005	0.0006	0.0009	0.0009
35	Ammonium molybdate	130,000		0.0005	0.0006	0.0005
36	Copper sulfate	12,600	0.0084	0.0099	0.0114	0.0113
37	Ferrous sulfate	3,000	0.0028	0.0034	0.0036	0.0035
40	Manganese sulfate	19,000	0.0023	0.0028	0.0029	0.0029
41	Zinc sulfate heptahydrate	6,240	0.0025	0.0030	0.0032	0.0031
	Cost (R$)		**8,2604**	**12.3355**	**11,5092**	**12,1084**

(a) Carry out fertigation three times a week, observing the compatibility between the fertilizers used.
(b) Days after transplanting seedlings to an agricultural greenhouse.
(c) Quantity of nutrients, in kilograms, per period, for 1000 tomato plants.

Source: Prepared by the author on the basis of research data.

Considering that the nutrients described in Table 2 are available in more than one fertilizer, a total of 43 possible sources to choose from, it is possible to replace a given nutrient source, on the basis of the fertilization recommendation, the characteristics of each fertilizer and the absorption by the plants, with another that can meet the nutritional needs of a crop but at a lower price, thus allowing a reduction in the total cost related to plant nutrition.

On the basis of the fertilization recommendations and nutrient sources recommended by the Bulletin of the Agronomic Institute of Campinas (IAC) for protected environments (TRANI, 2015), the cost of the fertilizer mixture for the crop's production cycle, a period of 150 days, was calculated, totaling R$839.50, according to the summary presented in Table 5. To calculate the total cost, the fertigation quantities were determined in each period, according to the recommendation of carrying out three fertigations per week, totaling 58 applications during the production cycle (150 days) of the crop in question.

Table 5. Costs of fertilizers used in the fertigation of table tomato crops[a] according to the fertilization recommendations of the IAC Bulletin.

Period DAT[c]	**Unit cost** (R$ kg^{-1})	**Fertigation**[b] (Amount)	**Total cost** (R$)
10 to 35 days	8,219	11	88,068
36 to 60 days	13,417	10	138,005
61 to 90 days	14,589	12	181,328
91 to 150 days	17,089	25	432,097
Total			**839,498**

(a) Values calculated considering a total of 1000 tomato plants, production cycle of 150 days or 21 weeks, with three weekly fertigations.
(b) Carry out fertigation three times a week, observing the compatibility between the fertilizers used.
(c) Days after transplanting seedlings to an agricultural greenhouse.

The application of the mathematical model made it possible to reduce the costs described in Table 5 by R$ 174.90, making the cost of the initial application recommendation 21% lower, as summarized in Table 6. This was possibly due to the diversification of fertilizers chosen by the model, which identified alternative sources of nutrients but with the same fertilization recommendation; that is, the nutritional needs of the plant were met at a lower cost (Table 6).

Table 6. Results obtained for the costs of fertilizers used in the fertigation of table tomato crops [a], according to the fertilization recommendations of the IAC Bulletin on the basis of the application of the mathematical model.

Period DAT[c]	**Unit cost** (R$ kg^{-1})	**Fertigation**[b] (Amount)	**Total cost** (R$)
10 to 35 days	8,260	11	88,504
36 to 60 days	12,335	10	126,879
61 to 90 days	11,509	12	143,043
91 to 150 days	12,1084	25	306,169
Total			**664,597**

(a) Values calculated considering a total of 1000 tomato plants, production cycle of 150 days or 21 weeks, with three weekly fertigations.
(b) Carry out fertigation three times a week, observing the compatibility between the fertilizers used.
(c) Days after transplanting seedlings to an agricultural greenhouse.

By identifying the optimal fertilizer mixture, the model meets the previously established constraints among the sources to be selected. When fertilizers are applied via fertigation, compatibility, electrical conductivity, and solubility are prerequisites for proper functioning of the entire fertilizer distribution process, making them essential.

Otherwise, damage to the distribution system can occur, resulting in financial losses for the producer.

Thus, it is worth highlighting that, unlike the research presented by Tieppo et al. (2010), Moreira-Barradas. Matula and Dolezal (2012), Pagán et al . (2015), Bueno-Delgado (2016), Almeida et al. (2016), Gallardo et al . (2016) and Pérez-Castro et al. (2017), the proposed methodology includes restrictions related to the electrical conductivity of the solution, the compatibility between fertilizers and the solubility, which eliminates the need for additional analyses after the implementation of the model. In other words, the proposed mathematical model meets the restrictions recommended in fertigation for calculating the amount of fertilizers at a low cost, making it more complete and efficient.

5 CONCLUSION

The proposed optimization model enables a reduction in the cost of fertilizer mixing, which results in a reduction in the total production cost.

The developed mathematical model made it possible to calculate fertilizer quantities, meeting the recommended restrictions on fertigation in terms of compatibility, electrical conductivity and solubility at a minimum cost.

Therefore, the methodology proposed in this research presents itself as an important tool to assist in the adequate management of fertigation.

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